Journal of Public Economics 90 (2006) 1879–1901

JOURNAL OF PUBLIC ECONOMICS

www.elsevier.com/locate/econbase

A general formula for the optimal level of social insurance ☆

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Received 16 May 2005; received in revised form 4 December 2005; accepted 2 January 2006
Available online 30 March 2006

Abstract

In an influential paper, Baily (1978) showed that the optimal level of unemployment insurance (UI) in a stylized static model depends on only three parameters: risk aversion, the consumption-smoothing benefit of UI, and the elasticity of unemployment durations with respect to the benefit rate. This paper examines the key economic assumptions under which these parameters determine the optimal level of social insurance. I show that a Baily-type expression, with an adjustment for precautionary saving motives, holds in a general class of dynamic models subject to weak regularity conditions. For example, the simple reduced-form formula derived here applies with arbitrary borrowing constraints, durable consumption goods, private insurance arrangements, and search and leisure benefits of unemployment.

JEL classification: H5; J65
Keywords: Unemployment insurance; Consumption-smoothing

1. Introduction

As social insurance programs grow rapidly in developed economies, a large literature assessing the economic costs and benefits of programs such as unemployment and disability insurance has emerged. The canonical normative analysis of social insurance is due to Baily (1978). Baily analyzes a stylized model of unemployment and obtains a simple inverse-elasticity formula for the optimal unemployment insurance (UI) benefit rate in terms of three parameters:

☆ I have benefited from comments by Alan Auerbach, Tom Crossley, Jon Gruber, Adam Looney, Emmanuel Saez, Ivan Werning, two anonymous referees, and seminar participants at the NBER Summer Institute. I thank the National Science Foundation and the Robert D. Burch Center for Tax Policy and Public Finance for financial support.
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doi:10.1016/j.jpubeco.2006.01.004

(1) the elasticity of unemployment durations with respect to benefits, which captures the moral hazard cost of benefit provision due to behavioral response; (2) the drop in consumption as a function of UI benefits, which quantifies the consumption-smoothing benefits; and (3) the coefficient of relative risk aversion (γ), which reflects the value of having a smoother consumption path. Guided by the intuition that these parameters are central in assessing the welfare consequences of unemployment insurance, many papers have estimated the effect of UI benefits on durations (e.g. Moffitt, 1985; Meyer, 1990) and consumption (e.g. Gruber, 1997; Browning and Crossley, 2001).

Since Baily's contribution, several studies have observed that his framework is restrictive and argued that the optimal level of social insurance differs under alternative assumptions. Examples include models with borrowing constraints (Flemming, 1978; Crossley and Low, 2005), more general search technologies (Lentz, 2004), and human capital accumulation effects (Brown and Kaufold, 1988). These papers derive formulas for the optimal benefit level in terms of the primitive structure of the model, and show that changing these primitives can have quantitatively large effects on optimal benefit rates in simulations. More recently, Golosov and Tsyvinski (2005) show that the welfare gain from government intervention is greatly reduced in models that allow for private insurance markets. Other studies have remarked on the limits of Baily's results less formally. Feldstein (2005) notes that calculations of optimal UI based on Baily's formula could be misleading because they do not adequately account for savings responses, while Gruber (1997) calibrates Baily's formula and cautions that the introduction of leisure benefits of unemployment could potentially change his results.

While these studies have identified several important factors in the analysis of social insurance, they have not attempted to obtain a reduced-form expression for the optimal benefit level based on observable elasticities (rather than deep primitives) in the more general setting that they consider. This paper investigates the key economic assumptions necessary to obtain such a formula.

I study a dynamic model where agents choose consumption, unemployment durations, and M other behaviors, such as spousal labor supply or human capital decisions, that enter a general time-separable utility function. Agents face a budget constraint and N other constraints, such borrowing or hours constraints, when choosing these behaviors. An arbitrary stochastic process determines the agent's employment status at each time. The model abstracts from the effects of UI on firm behavior by assuming that the supply of jobs and wage rates are not endogenous to the benefit level.[1]

The main result is that Baily-type expressions for both the optimal benefit level and the marginal welfare gain from an increase in social insurance apply much more generally than suggested by the existing literature.[2] In particular, suppose each constraint on consumption while unemployed can be loosened by raising benefits, and each constraint on consumption while employed can be loosened by reducing the UI tax. As discussed below, virtually any economically plausible constraint in a model where income streams are fungible satisfies this requirement. Then, under some weak regularity conditions that make the government's optimization problem well behaved, the optimal benefit rate is approximately determined by

[1] Because of this limitation, the formula derived in this paper does not apply to the recent equilibrium models of UI analyzed by Acemoglu and Shimer (1999) and others.

[2] Although the model analyzed here refers to an unemployment shock, with a change of notation, the general case can be used to model social insurance against other shocks such as injury or disability. In this sense, the formula derived here is informative about optimal state-contingent redistributive policies in general and not just unemployment insurance.

the same three parameters described above, along with the coefficient of relative prudence. The approximation requires that fourth and higher-order terms of utility over-consumption are small; calibrations with power utility functions indicate that the error associated with this approximation is on the order of 2–4%. When the third-order terms of utility are small as well (i.e., when agents do not have precautionary savings motives), Baily's three-parameter formula carries over directly to the general case.

These results show that calculations of the optimal benefit rate based on reduced-form empirical estimates are valid in much broader environments than earlier studies have suggested. For example, the simple formulas derived here hold even with arbitrary borrowing constraints, endogenous insurance through channels such as spousal labor supply, leisure benefits of unemployment, portfolio choice, durable goods, and human capital decisions. Variations in the structure of the underlying model do not affect the formula because the four primary inputs are sufficient statistics for the purpose of computing the optimal benefit level in a general environment.

The converse of this result is that the optimal benefit rate does *not* explicitly depend on several other parameters that one intuitively expects should matter. For example, factors such as the leisure benefits of unemployment or the potential role of UI in improving job matches by subsidizing search seem to play no role in the calculation of the optimal benefit level. In addition, the relative magnitudes of income and substitution effects in the link between UI and durations appear to be irrelevant.

The second part of this paper explores why the formula exhibits these features. The basic reason is that the elasticities that enter the formula are all functions of other aspects of the agent's behavior and preferences. For instance, if unemployment has large leisure benefits, agents would elect to have a longer duration and therefore a larger consumption drop, ultimately leading to a higher optimal benefit rate, as one would expect. The formula presented below is thus only one representation of a reduced-form expression for optimal benefits. To illustrate why the restrictions implied by different representations of the formula matter, I analyze how income and substitution effects in unemployment durations relate to the optimal benefit level in greater detail. Using a Slutsky decomposition, I show that γ (risk aversion) is pinned down by the ratio of the income elasticity to the substitution elasticity. Large income effects imply higher risk aversion and therefore generate a higher optimal benefit rate, as one would expect given that income effects are non-distortionary. However, *conditional* on the value of γ and the other three primary inputs, the magnitudes of the income effect is irrelevant.

This point reveals an important tradeoff in evaluating policies using the formula proposed here. The power of this reduced-form approach is that it does not require complete specification of the underlying model, permitting an analysis that is not sensitive to specific modelling choices. The danger is that one might choose elasticities that are inconsistent with each other or with other behavioral responses. In the income effects example, one might calibrate the formula with a low risk aversion parameter (as in certain cases considered by Baily, 1978 and Gruber, 1997), failing to recognize that this would contradict empirical studies that have identified large income effects on labor supply for the unemployed (e.g. Mincer, 1962; Cullen and Gruber, 1998; Chetty, 2005). This inconsistency is not immediately apparent because the set of primitives generating the high-level elasticities is never explicitly identified. Hence, while the formula for optimal social insurance derived here is widely applicable, it should ideally be implemented with support from empirical estimates of other behavioral responses coupled with structural tests for consistency of the various parameters.

The remainder of the paper is organized as follows. The next section derives formulas for the optimal benefit level and the welfare gain from raising benefits in a stylized model to build intuition. Section 3 shows that these formulas carry over with small modifications to the general case. Section 4 turns to the counterintuitive features of the result, demonstrating in particular how the size of income and substitution effects matter. The final section offers concluding remarks.

2. The optimal UI benefit level

I consider the optimal benefits problem in a model where agents receive a constant unemployment benefit of b while unemployed.[3] The government finances the benefits by levying a lump-sum tax of τ on employed agents. The lump-sum tax assumption simplifies the algebra, and also has the virtue of describing actual practice. In the United States, UI benefits are financed by a payroll tax applied only to the first \$10,000 of income, and is thus inframarginal (and effectively a lump-sum tax) for most workers.

I make three substantive assumptions throughout the analysis. First, I take wages as fixed, ignoring the possibility that UI benefits have general-equilibrium effects by changing the supply and demand for jobs with different risk characteristics. Second, I abstract from distortions to firm behavior (e.g., those caused by imperfect experience rating) by assuming that expected unemployment durations are fully determined by workers who take their tax burden as fixed. Finally, I assume that agents' choices have no externalities, so that all private and social marginal costs are equal in the absence of a government UI system. For example, spillovers in search behavior and distortions in the economy that create wedges between private and social marginal costs are ruled out.

2.1. A special case: tenure review

I begin with a stylized model where the derivation of the optimal benefit rate (b^*) is most transparent. This model should not be viewed as a realistic depiction of the UI problem since it ignores important features such as search behavior under uncertainty and borrowing constraints while unemployed. Despite these limitations, the simple model is informative because the formula for the optimal benefit rate in a more realistic and general environment ends up being quite similar.

Consider an environment where agents face unemployment risk at only one time in their lives. For concreteness, it is helpful to think of this model as an analysis of optimal unemployment insurance for academics being reviewed for tenure. Suppose a representative assistant professor arrives at his tenure review (time 0) with assets A_0. He lives for one unit of time after the review, until $t=1$. The agent is informed of the tenure decision at $t=0$, at which point he either gets a permanent job that pays a wage of w (probability $1-p$) or is denied tenure and becomes unemployed (probability p). Assume for now that p is exogenous and does not vary with the benefit level. In the employed (tenured) state, there is no risk of job loss until death, and the agent makes no labor supply choices. In the unemployed state (where tenure has been denied), the agent must search for a new job. Assume that the agent can control his

[3] The optimal path or duration of benefits, which has attracted much attention in recent work (see e.g., Davidson and Woodbury, 1997; Hopenhayn and Nicolini, 1997; Shimer and Werning, 2005), is outside the scope of this paper.

unemployment duration, D, deterministically by varying search effort. Search costs, the leisure value of unemployment, and the benefits of additional search via improved job matches are captured by a concave, increasing function $\psi(D)$.

The only constraints are the budget constraint in each state. Assume for simplicity that the UI tax τ is collected only in the tenured state, so that the agent has to pay no taxes while working in a new job if he lost his job at $t=0$. Normalize the interest rate and discount rate at 0. Since there is no uncertainty, discounting, or income growth after the tenure decision is known, the optimal consumption path is flat in both states. Let c_{e} denote consumption in the tenured state and c_{u} denote consumption in the untenured state. Let $u(c)$ denote utility over consumption, which I assume is strictly concave and state-independent. The agent's problem at time 0 is thus to choose c_{e}, c_{u}, and D to

$$\begin{aligned}
&\max(1-p)u(c_{\mathrm{e}})+p\{u(c_{\mathrm{u}})+\psi(D)\}\\
&\text{s.t. } A_0+(w-\tau)-c_{\mathrm{e}}\geq 0\\
&A_0+bD+w(1D)-c_{\mathrm{u}}\geq 0
\end{aligned}$$

Let $V(b,\tau)$ denote the solution to this problem for a given unemployment benefit b and UI tax τ. The benevolent social planner's problem is to choose the benefit rate and UI tax pair $\{b,\ \tau\}$ that maximizes the agent's indirect utility subject to the balanced-budget constraint for the UI system (taxes collected equal benefits paid in expectation):

$$\begin{aligned}
&\max_{b,\tau} V(b,\tau)\\
&\text{s.t.}(1-p)\tau=pbD
\end{aligned}$$

The following proposition gives two approximate solutions to this problem. Note that this and subsequent results about the optimal benefit rate characterize b^* when it is positive. When this condition has a positive solution, that solution is a global maximum. When there is no solution to the equation that defines b^* at an interior optimum, it follows that $b^*=0$ under the regularity conditions used to ensure strict concavity of $V(b)$.

Proposition 1. *If the third and higher order terms of $u(c)$ are small ($u'''(c)\approx 0$), the optimal benefit rate b^* is implicitly defined by*

$$\gamma\frac{\Delta c}{c}(b^*)\approx\varepsilon_{D,b} \tag{1}$$

If the fourth and higher order terms of $u(c)$ are small ($u(c)\approx 0$), b^* is defined by

$$\gamma\frac{\Delta c}{c}(b^*)\left[1+\frac{1}{2}\rho\frac{\Delta c}{c}(b^*)\right]\approx\varepsilon_{D,b} \tag{2}$$

where $\frac{\Delta c}{c}=\frac{c_{\mathrm{e}}-c_{\mathrm{u}}}{c_{\mathrm{e}}}$=consumption drop due to unemployment; $\gamma=-\frac{u''(c_{\mathrm{e}})}{u'(c_{\mathrm{e}})}c_{\mathrm{e}}$=coefficient of relative risk aversion; $\rho=-\frac{u'''(c_{\mathrm{e}})}{u''(c_{\mathrm{e}})}c_{\mathrm{e}}$=coefficient of relative prudence; $\varepsilon_{D,b}=\frac{\mathrm{dlog}D}{\mathrm{dlog}b}$=elasticity of duration w.r.t. benefits.

Proof . At an interior optimum, the optimal benefit rate must satisfy

$$\mathrm{d}V/\mathrm{d}b(b^*)=0$$

where $\mathrm{d}V/\mathrm{d}b$ denotes the total derivative of V w.r.t. b, recognizing that τ is a function of b determined by the government's budget constraint. To calculate $\mathrm{d}V/\mathrm{d}b$, note first that $V(b)$ can be written as

$$\begin{aligned} V(b) = \max_{c_{\mathrm{e}},c_{\mathrm{u}},D,\lambda_{\mathrm{e}}\lambda_{\mathrm{u}}} & (1-p)u(c_{\mathrm{e}}) + p\{u(c_{\mathrm{u}}) + \psi(D)\} + \lambda_{\mathrm{e}}[A_0 + (w-\tau) - c_{\mathrm{e}}] \\ & + \lambda_{\mathrm{u}}[A_0 + bD + w(1-D) - c_{\mathrm{u}}] \end{aligned}$$

where λ_{e} and λ_{u} are the LaGrange multipliers that give the marginal value of relaxing the budget constraint while employed and unemployed. Since this function has already been optimized over $\{c_{\mathrm{e}}, c_{\mathrm{u}}, D, \lambda_{\mathrm{e}}, \lambda_{\mathrm{u}}\}$, changes in these variables do not have first-order effects on V (an application of the Envelope Theorem). Hence,

$$\mathrm{d}V/\mathrm{d}b(b^*) = -\lambda_{\mathrm{e}}\frac{\mathrm{d}\tau}{\mathrm{d}b} + \lambda_{\mathrm{u}}D = 0 \tag{3}$$

$$\Rightarrow \lambda_{\mathrm{e}}\frac{\mathrm{d}\tau}{\mathrm{d}b} = \lambda_{\mathrm{u}}D. \tag{4}$$

Agent optimization implies that the multipliers are equal to the marginal utility of consumption in each state:

$$\lambda_{\mathrm{e}} = (1-p)u'(c_{\mathrm{e}}) \tag{5}$$

$$\lambda_{\mathrm{u}} = pu'(c_{\mathrm{u}}) \tag{6}$$

The government's UI budget constraint implies

$$\frac{\mathrm{d}\tau}{\mathrm{d}b} = \frac{p}{1-p}\left[D + b\frac{\mathrm{d}D}{\mathrm{d}b}\right]$$

and plugging these expressions into (3) and simplifying yields

$$u'(c_{\mathrm{e}})\left[1 + \frac{b}{D}\frac{\mathrm{d}D}{\mathrm{d}b}\right] = u'(c_{\mathrm{u}}) \tag{7}$$

This optimality condition captures a basic intuition that carries over to the general case: The optimal level of benefits offsets the marginal benefit of raising consumption by \$1 in the untenured state (RHS of (7)) against the marginal cost of raising the UI tax in the tenured state to cover the required increase in the UI benefit (LHS of (7)). The marginal cost of raising the UI tax to finance a \$1 increase in c_{u} is given by the direct cost $u'(c_{\mathrm{e}})$ plus an added term arising from the agent's behavioral response of extending his unemployment duration, which acts to reduce c_{u}.[4]

[4] If the UI tax were collected in both the tenured and untenured states, the $u'(c_{\mathrm{e}})$ term in (7) would be replaced by an average of marginal utilities over the times when the agent is employed, as in the general case analyzed below.

Rearranging (7) gives

$$\frac{u'(c_u) - u'(c_e)}{u'(c_e)} = \frac{b}{D}\frac{dD}{db} \tag{8}$$

This equation provides an exact definition for the optimal benefit rate, and can be solved for b^* by choosing a function form for u. An approximate solution can be obtained by simplifying the left-hand side of this expression using a Taylor expansion to write

$$u'(c_u) - u'(c_e) \approx u''(c_e)(c_u - c_e) + \frac{1}{2}u'''(c_e)(c_e - c_u)^2.$$

Using the definitions of γ and ρ, we obtain

$$\frac{u'(c_u) - u'(c_e)}{u'(c_e)} \approx -\frac{u''}{u'}c_e\frac{\Delta c}{c} + \frac{1}{2}\frac{u'''}{u''}c_e\frac{u''}{u'}c_e\left(\frac{\Delta c}{c}\right)^2$$

$$= \gamma\frac{\Delta c}{c} + \frac{1}{2}\rho\gamma\left(\frac{\Delta c}{c}\right)^2. \tag{9}$$

Plugging this expression into the left-hand side of (8) and factoring yields the formula given in (2). Note that $u'''=0 \Rightarrow \rho=0$, in which case (2) reduces to (1).

To prove that b^* is a global maximum, one can show that $\frac{d^2V}{db^2}<0$. This condition is established under certain regularity conditions for the general case below. □

The first approximate solution for b^* given in Proposition 1 is the same as Baily's (1978) formula. He ignores third-order terms of u in his derivation, effectively assuming that precautionary savings motives are small, in which case utility is well approximated by a quadratic function. Unfortunately, the approximation error induced by ignoring the third-order terms in this case is sometimes large. In particular, using power (CRRA) utility with γ ranging from 1 to 5, the $\frac{\Delta c}{c}(b)$ function as given by Gruber's (1997) estimates, and $\varepsilon_{D,b}=0.5$, Baily's approximate solution sometimes underestimates the exact b^* by more than 30%. To obtain a more precise solution, the effects of third-order terms in u must be taken into account. This yields the formula in (2), which has an additional coefficient of relative prudence term. This formula, which assumes that the fourth and higher-order terms of u are small, is a much more successful approximation: the difference between the exact and approximate b^* is always less than 4% for the calibration exercises described above. Hence, using an estimate of the reduced-form relationship between $\frac{\Delta c}{c}$ and b, one can obtain a reasonably good estimate of the optimal b^* by solving (2) for b.[5]

It is helpful to remark on the mechanism underlying Proposition 1 since it carries over to the general case. At a mathematical level, the basic idea is to exploit the envelope condition, which permits us to write the marginal value of raising b purely in terms of the multipliers λ_u and λ_e. Agent optimization then allows us to express λ_u and λ_e in terms of the marginal utilities of consumption in each state, as in (5) and (6). Intuitively, the results arise because the agent has already equated all marginal utilities within each state at the optimum. Therefore, we can assume that extra benefits are spent on solely on c_u (and that higher taxes are financed solely by reducing

[5] One can of course formulate examples where even the third-order approximation will not work well. If one has strong priors about the fourth-order terms of u, they can be used to obtain a more precise formula for b^* by expanding the Taylor series in (9) by one more term.

c_e) when computing welfare changes. This allows us to write the welfare change purely in terms of $u'(c_u)$ and $u'(c_e)$ and ignore all behavioral responses when calculating b^* except for the $\varepsilon_{D,b}$ parameter that enters the government's budget constraint directly. The next section shows that an envelope condition can be applied to obtain a similar formula for b^* in a more general environment.

3. The general case

3.1. Choice variables

Consider a continuous-time dynamic model where a representative agent faces persistent unemployment risk. Normalize the length of life to be one unit, so that time $t \in [0,1]$. Let ω_t denote a state variable that contains the information from the agent's history up to time t relevant in determining period t employment status and behavior. For example, ω_t may include prior employment records, which determine current employment status and future job layoff probabilities. For notational simplicity, it is convenient to assume that ω_t is a scalar, but all of the results that follow hold if ω_t were a vector. The evolution of ω_t is determined by an arbitrary stochastic process. Let $F_t(\omega_t)$ denote the unconditional distribution function of ω_t given information available at time 0. Assume that F_t is a smooth function and let Ω denote the maximal support of F_t.

The agent chooses behavior at each time t contingent on the value of ω_t. Let $c(t,\omega_t)$ denote consumption at time t in state ω_t. The agent also chooses a vector of M other behaviors in each state: $x(t,\omega_t)=(x^1(t, \omega_t), \ldots, x^M(t, \omega_t))$. These could include choices such as search effort while unemployed, reservation wage while unemployed, level of work effort (or shirking) while employed, private insurance purchases, amount of borrowing from friends, portfolio choice, human capital investments, etc. Assume that utility is time-separable and let $u(c(t,\omega_t), x(t,\omega_t))$ denote the felicity utility of the agent as a function of his choices. I assume that utility is state-independent, i.e. that the marginal utility of consumption is determined purely by the current level of consumption and not whether the agent is currently employed or unemployed.[6] Let $c=\{c(t,\omega_t)\}_{t\in[0,1],\omega_t\in\Omega}$ and $x=\{x(t,\omega_t)\}_{t\in[0,1],\omega_t\in\Omega}$ denote the full program of state-contingent choices over life.

Let $\theta(t, \omega_t, c, x)$ denote an agent's employment status at time t in state ω_t. If $\theta=1$, the agent is employed, and if $\theta=0$, the agent is unemployed. Since θ is an arbitrary function of ω_t, which is a random variable, the model allows for uncertainty in unemployment duration lengths. I allow θ to be a function of c and x because the agent's choices (e.g. search effort or savings behavior) may affect his job search decisions and therefore his employment status.

Define $D(c, x)$ as the expected fraction of his life that the agent spends unemployed given a program (c, x). Note that this and all subsequent expectations are taken over all times *and* all states (histories up to t):

$$D(c,x) = E[1-\theta(t,\omega_t)] = \int_0^1 \int_{\omega_t\in\Omega} [1-\theta(t,\omega_t,c,x)]\mathrm{d}F_t(\omega_t)\mathrm{d}t$$

To reduce notation, the arguments of θ and D are sometimes suppressed below when the meaning is not ambiguous.

[6] If utility is state-dependent, the arguments below go through, except that the final Taylor approximation for the difference in marginal utilities in terms of the average consumption drop requires an adjustment for state dependence.

Let $\overline{c_e}$ and $\overline{c_u}$ denote mean consumption while employed and unemployed, respectively:

$$\overline{c_e} = E[c(t,\omega_t)|\theta(t,\omega_t)=1] = \frac{\int_t \int_{\omega_t} \theta(t,\omega_t)c(t,\omega_t)\mathrm{d}F_t(\omega_t)\mathrm{d}t}{\int_t \int_{\omega_t} \theta(t,\omega_t)\mathrm{d}F_t(\omega_t)\mathrm{d}t}$$

$$\overline{c_u} = E[c(t,\omega_t)|\theta(t,\omega_t)=0] = \frac{\int_t \int_{\omega_t} (1-\theta(t,\omega_t))c(t,\omega_t)\mathrm{d}F_t(\omega_t)\mathrm{d}t}{\int_t \int_{\omega_t} (1-\theta(t,\omega_t))\mathrm{d}F_t(\omega_t)\mathrm{d}t}.$$

3.2. Constraints

The agent faces a standard dynamic budget constraint. Income is a function of his current employment state: he earns $w-\tau$ if employed, and UI benefits of b if unemployed. Income may also be earned from other sources (e.g. borrowing or by adding a second earner). The effects of these other behaviors on income at time t is captured through an arbitrary function $f(x(t,\omega_t))$.

$$\dot{A}(t,\omega_t) = f(x(t,\omega_t)) + \theta(t,\omega_t)(w-\tau) + (1-\theta(t,\omega_t))b - c(t,\omega_t)\forall t,\omega_t \tag{10}$$

There is also a terminal condition which requires that the agent maintain assets above some bound in all states of the terminal period:

$$A(1,\omega_1) \geq A_{\mathrm{term}} \qquad \forall \omega_1$$

The agent faces a set of N additional constraints in each state ω_t at each time t

$$g_{i\omega t}(c,x;b,\tau) \geq \bar{k}_{i\omega t}, \qquad i=1,..,N.$$

Let $\lambda_{\omega,t}$ denote the multiplier on the dynamic budget constraint in state ω_t at time t; $\lambda_{\omega 1,T}$ the multipliers on the terminal conditions; and $\lambda_{gi,\omega,t}$ the multipliers on the additional constraints. Each of these multipliers equal the marginal value of relaxing the corresponding constraint in the optimal program.

Agent's and planner's problems. The agent chooses a program (c, x) to

$$\begin{aligned}\max \int_t \int_{\omega_t} & u(c(t,\omega_t),x(t,\omega_t))\mathrm{d}F_t(\omega_t)\mathrm{d}t + \int_{\omega_1} \lambda_{\omega_1,T}(A(1,\omega_1) - A_{\mathrm{term}})\mathrm{d}F_1(\omega_1) \\ &+ \int_t \int_{\omega_t} \lambda_{\omega,t}\{f(x(t,\omega_t)) + \theta(t,\omega_t)(w-\tau) + (1-\theta(t,\omega_t))b - c(t,\omega_t)\}\mathrm{d}F_t(\omega_t)\mathrm{d}t \\ &+ \sum_{i=1}^{N} \int_t \int_{\omega_t} \lambda_{g_i,\omega,t}\left(g(c,x;b,\tau) - \bar{k}_{i\omega t}\right)\mathrm{d}F_t(\omega_t)\mathrm{d}t.\end{aligned}$$

Let $V(b,\tau)$ denote the maximal value for this problem for a given unemployment benefit b and tax rate τ. The social planner's problem is to

$$\max_{b,\tau} V(b,\tau)$$

subject to balancing the government budget in expectation, which requires:

$$\tau \int_t \int_{\omega_t} \theta(t) \mathrm{d}F_t(\omega_t)\mathrm{d}t = b \int_t \int_{\omega_t} [1-\theta(t)]\mathrm{d}_t(\omega_t)\mathrm{d}t$$

$$\Rightarrow \tau(1-D) = Db.$$

Ensuring that the solution to the social planner's problem can be obtained from first-order conditions requires some regularity assumptions, which are specified below.

Assumption 1. Total lifetime utility ($\int_t \int_{\omega_t} u(c(t,\omega_t), x(t,\omega_t))\mathrm{d}F_t(\omega_t)\mathrm{d}_t$) is smooth, increasing, and strictly quasiconcave in (c,x).

Assumption 2. The set of choices $\{(c,x)\}$ that satisfy all the constraints is convex.

Assumption 3. In the agent's optimal program, the set of binding constraints does not change for a perturbation of b in some open interval $(b-\varepsilon, b+\varepsilon)$.

Assumptions 1 and 2 guarantee that the agent's problem has a unique global constrained maxima (c,x). Together with Assumption 3, these assumptions imply that the Envelope Theorem can be applied to obtain $\frac{\mathrm{d}V}{\mathrm{d}b}$ (see the mathematical appendix in Mas-Colell (1995) for a proof). Without loss of generality, assume below that all of the auxiliary g constraints are binding; any constraint that is slack can be ignored under the third assumption.

The following set of conditions are sufficient (but not necessary) to establish that $V(b)$ is a strictly concave function, which ensures that any b satisfying the first-order condition is a global maximum.

Assumption 4. Consumption while unemployed is weakly increasing in b; consumption while employed is weakly decreasing in τ; and the marginal increment in τ required to finance an increase in b is weakly increasing in b:

$$\frac{\partial \bar{c}_{\mathrm{u}}}{\partial b} \geq 0, \quad \frac{\partial \bar{c}_{\mathrm{e}}}{\partial \tau} \leq 0, \quad \frac{\mathrm{d}^2\tau}{\mathrm{d}b^2} \geq 0$$

The first two parts of this assumption essentially require that the direct effect of changes in the UI tax and benefits are not swamped by behavioral responses in the opposite direction. The third part requires that the marginal cost of raising funds to finance UI is increasing. To understand this condition, observe that $\frac{\mathrm{d}\tau}{\mathrm{d}b} = \frac{D}{1-D}\left[1+\frac{\varepsilon_{D,b}}{1-D}\right]$. Given that $\mathrm{d}D/\mathrm{d}b > 0$, it follows that $\frac{\mathrm{d}^2\tau}{\mathrm{d}b^2} > 0$ in the benchmark case where the duration elasticity of UI benefits is constant $\left(\frac{\partial \varepsilon_{D,b}}{\partial b} = 0\right)$. Higher benefits raise the fraction of the time the agent is unemployed, shrinking the UI tax collection base while expanding the length of time that the agent receives benefits. A marginal increase in b therefore requires a larger increase in τ to balance the budget when b is high to begin with. $\frac{\partial^2\tau}{\partial b^2}$ could only be negative if $\varepsilon_{D,b}$ falls sharply as b rises $\left(\frac{\partial \varepsilon_{D,b}}{\partial b} 0\right)$, swamping the direct effect due to changes in D. Estimates of $\varepsilon_{D,b}$ are broadly similar across studies with different levels of benefit generosity, suggesting that $\varepsilon_{D,b}$ does not vary sharply with b. Hence, under most plausible scenarios, the formulas given here are necessary and sufficient conditions for b*.

3.3. Consumption-UI constraint condition

The derivation for the static model shows that we must be able to quantify the costs and benefits of unemployment insurance solely through the marginal utilities of consumption in each state to obtain a simple formula for b^*. This is feasible if higher benefits relax all constraints on consumption while unemployed and higher taxes tighten all constraints on consumption while employed. Intuitively, as long as extra benefits can be spent on raising consumption while unemployed, we can assume for the purposes of welfare calculations that the agent will do this at the margin. This will permit us to write the benefits of UI purely in terms of the average marginal utilities of consumption. The following assumption states the necessary restrictions on the constraints formally.

Assumption 5. The feasible set of choices can be defined using a set of constraints $\{g_{i\omega_t}\}$ such that $\forall i \forall t \forall \omega_t$

$$\frac{\partial g_{i\omega t}}{\partial b} = -(1-\theta(t,\omega_t))\frac{\partial g_{i\omega t}}{\partial c(t,\omega_t)}$$
$$\frac{\partial g_{i\omega t}}{\partial \tau} = \theta(t,\omega_t)\frac{\partial g_{i\omega t}}{\partial c(t,\omega_t)}$$
$$\frac{\partial g_{i\omega t}}{\partial c_\theta(s,\omega_s)} = 0 \qquad \text{if } t \neq s.$$

Assumption 5 requires that the set of binding constraints can be written so that at all times (a) the UI benefit and consumption while unemployed enter each constraint in the same way, (b) the UI tax and consumption while employed enter each constraint in the same way, and (c) consumption at two different times s and t do not enter the same constraint together. It is helpful to illustrate when this condition holds with some examples:

(a) Budget constraints. In the simplest model, the only constraint is the budget constraint. To verify that the dynamic budget constraint in (10) satisfies Assumption 5, note that $\frac{\partial \dot{A}}{\partial b} = -\frac{\partial \dot{A}}{\partial c(t,\omega_t)} = 1$ if $\theta(t,\omega_t)=0$ and $\frac{\partial \dot{A}}{\partial \tau} = \frac{\partial \dot{A}}{\partial c(t,\omega_t)} = -1$ if $\theta(t,\omega_t)=1$. In addition, only $c(t,\omega_t)$ appears in each constraint at time t. Hence, Assumption 5 is satisfied, explaining why (2) was obtained in the static case.

(b) Borrowing constraint if unemployed:

$$g_{1\omega t} = (1-\theta(t,\omega_t))(A(t,\omega_t)+b-c(t,\omega_t)) \geq 0$$

If this constraint binds when $\theta=0$, $\frac{\partial g_{1\omega t}}{\partial b} = -\frac{\partial g_{1\omega t}}{\partial c(t,\omega_t)} = 1$ and $\frac{\partial g_{1\omega t}}{\partial \tau} = \frac{\partial g_{1\omega t}}{\partial c(t,\omega_t)} = 0$, so Assumption 5 holds.

(c) Private insurance market. Suppose the agent holds a private insurance contract that charges a premium ρ_e when he is employed and has a net payout of ρ_u in the unemployed state. This adds a term $-\theta(t,\omega_t)\rho_e + (1-\theta(t,\omega_t)\rho_u$ to the dynamic budget constraint, implying that the derivatives of A in example (a) are unchanged and Assumption 5 still holds. Abstractly, private insurance arrangements change $f(x(t,\omega_t))$, with no consequence for how consumption and UI benefits/taxes enter the budget constraints.

(d) Hours constraint while employed. Suppose the agent is able to choose labor supply $l(t,\omega_t)$ on the intensive margin while employed but cannot work for more than H hours by law. Then he faces the additional constraint at all times t:

$$g_{2\omega t} = H - l(t,\omega_t) \geq 0$$

Since $\frac{\partial g_{2\omega t}}{\partial b} = \frac{\partial g_{2\omega t}}{\partial \tau} = \frac{\partial g_{2\omega t}}{\partial c(t,\omega_t)} = 0 \forall t \forall \omega_t$, Assumption 5 is satisfied for this constraint.

(e) Subsistence constraint. Suppose the agent must maintain consumption above a level c at all times:

$$g_{3\omega t} = c(t,\omega_t) - \underline{c} \geq 0 \forall \omega_t, t$$

If this constraint binds at t' for some ω', in that instance $\frac{\partial g_{3\omega' t'}}{\partial b} = 0 \neq \frac{\partial g_{3\omega' t'}}{\partial c(t', \omega'_t)} = 1$, so Assumption 5 is *not* satisfied here.

Although a subsistence constraint can technically violate the consumption-UI constraint condition, it represents a pathological case. Most agents are able to cut consumption when benefits are lowered in practice (Gruber, 1997). Moreover, such a constraint is unlikely to literally bind because one would expect the marginal utility of consumption to rise to infinity as consumption falls to c, preventing agents from reaching this point. More generally, as long as different sources of income are fungible, agents should be able to use higher benefits (or lower taxes) to change their consumption in the relevant state. The only reason this might not be feasible is because of technological constraints on consumption. Since most economically plausible constraints do not involve such restrictions, they are likely to satisfy the consumption-UI constraint condition.

Assumption 5 essentially guarantees that the marginal value of increasing benefits and raising the UI tax can be read directly from the average marginal utilities of consumption in each state. The following lemma establishes this connection.

Lemma 1. *The marginal value of increasing the UI benefit while balancing the UI budget is*

$$\mathrm{d}V/\mathrm{d}b = -\frac{\mathrm{d}\tau}{\mathrm{d}b}(1-D)Eu'(c_\mathrm{e}) + DEu'(c_\mathrm{u}) \tag{11}$$

where the average marginal utilities of consumption while employed and unemployed are

$$Eu'(c_\mathrm{e}) = \frac{\int_t \int_{\omega_t} \theta(t,\omega_t) u'(c(t,\omega_t)) \mathrm{d}F_t(\omega_t)\mathrm{d}t}{\int_t \int_{\omega_t} \theta(t,\omega_t)\mathrm{d}F_t(\omega_t)\mathrm{d}t}$$

$$Eu'(c_\mathrm{u}) = \frac{\int_t \int_{\omega_t} (1-\theta(t,\omega_t)) u'(c(t,\omega_t)) \mathrm{d}F_t(\omega_t)\mathrm{d}t}{\int_t \int_{\omega_t} (1-\theta(t,\omega_t))\mathrm{d}F_t(\omega_t)\mathrm{d}t}$$

Proof . Since behavioral responses to the change in benefits have no first-order effect on V (the envelope condition),

$$\begin{aligned} \mathrm{d}V/\mathrm{d}b = & -\frac{\mathrm{d}\tau}{\mathrm{d}b}\int_t \int_{\omega_t}\left[\theta(t,\omega_t)\lambda_{\omega,t} - \sum \lambda_{g_i,\omega,t}\frac{\partial g_{i\omega t}}{\partial \tau}\right]\mathrm{d}F_t(\omega_t)\mathrm{d}t \\ & + \int_t \int_{\omega_t}\left[(1-\theta(t,\omega_t))\lambda_{\omega,t} + \sum \lambda_{g_i,\omega,t}\frac{\partial g_{i\omega t}}{\partial b}\right]\mathrm{d}F_t(\omega_t)\mathrm{d}t \end{aligned} \tag{12}$$

Using the third part of Assumption 5, agent optimization requires that the marginal utility of consumption in each state can be written as a function of the corresponding multipliers at time t:

$$u'(c(t,\omega_t)) = \lambda_{\omega,t} - \sum \lambda_{g_i,\omega,t}\frac{\partial g_{i\omega t}}{\partial c(t,\omega_t)} \forall t, \omega_t \tag{13}$$

The first two parts of Assumption 5 imply that $\forall t,\ \omega_t$

$$\sum \lambda_{g_i,\omega,t} \frac{\partial g_{i\omega t}}{\partial \tau} = \sum \lambda_{g_i,\omega,t} \theta(t,\omega_t) \frac{\partial g_{i\omega t}}{\partial c(t,\omega_t)}$$

$$\sum \lambda_{g_i,\omega,t} \frac{\partial g_{i\omega t}}{\partial b} = -\sum \lambda_{g_i,\omega,t} (1-\theta(t,\omega_t)) \frac{\partial g_{i\omega t}}{\partial c(t,\omega_t)}$$

After plugging these expressions into (12) and factoring out the θ terms, we can substitute (13) into each integral to obtain

$$\begin{aligned} \mathrm{d}V/\mathrm{d}b = & -\frac{\mathrm{d}\tau}{\mathrm{d}b} \int_t \int_{\omega_t} \theta(t,\omega_t) u'(c(t,\omega_t)) \mathrm{d}F_t(\omega_t) \mathrm{d}t \\ & + \int_t \int_{\omega_t} (1-\theta(t)) u'(c(t,\omega_t)) \mathrm{d}F_t(\omega_t) \mathrm{d}t \end{aligned} \tag{14}$$

Substituting in the definitions of D and $Eu'(c_z)$ for $z \in (e,u)$ yields (11). □

Lemma 1 reflects the same basic intuition that underlies (7) in the static model. The marginal value of raising benefits by one dollar is the average marginal utility of consumption while unemployed times the amount of time unemployed less the marginal cost of raising those funds from the employed state. This marginal cost is given by the product of the average marginal utility of consumption while employed and $\frac{\mathrm{d}\tau}{\mathrm{d}b}$. To see why the consumption-UI constraint condition is needed to establish this result, consider an agent who faces a binding subsistence constraint while unemployed. This agent will continue to consume c when unemployed even if b is changed. Consequently, the marginal value of UI benefits cannot be directly inferred from the marginal utility of consumption, since the benefits are not used to raise consumption at the margin.

3.4. Approximation for average marginal utilities

It can be shown that b^* depends exactly on the difference in average marginal utilities between the employed and unemployed states, $Eu'(c_{\mathrm{u}}) - Eu'(c_{\mathrm{e}})$, under the preceding assumptions. However, it is convenient to identify conditions under which the average marginal utility in each employment state can be approximated by the marginal utility of average consumption in that state (i.e., when the order of integration can be switched). This is the purpose of the next result.

Lemma 2. *If the third and higher order terms of u are small ($u''' \approx 0$), the average marginal utility of consumption when employed (or unemployed) is approximately the marginal utility of consumption at* $\overline{c_{\mathrm{e}}}$ (or $\overline{c_{\mathrm{u}}}$)*:*

$$Eu'(c_{\mathrm{e}}) \approx u'(\overline{c_{\mathrm{e}}})$$

$$Eu'(c_{\mathrm{u}}) \approx u'(\overline{c_{\mathrm{u}}}) \tag{15}$$

If the fourth and higher order terms of u are small ($u''' \approx 0$),

$$Eu'(c_{\mathrm{e}}) \approx u'(\overline{c_{\mathrm{e}}})\left(1+\gamma\rho s_{\mathrm{e}}^2\right)$$

$$Eu'(c_{\mathrm{u}}) \approx u'(\overline{c_{\mathrm{u}}})\left(1+\gamma\rho s_{\mathrm{u}}^2\right) \tag{16}$$

where γ and ρ are defined as in Proposition 1 *and* $s_{\mathrm{e}} = \frac{\left[E(c(t,\omega_t)-\overline{c_{\mathrm{e}}})^2|\theta(t,\omega_t)=1\right]^{1/2}}{\overline{c_{\mathrm{e}}}}$ *is the coefficient of variation of consumption when employed and s_u is defined analogously when $\theta = 0$.*

Proof . Consider the employed case. Take a Taylor expansion of u around $\overline{c_{\mathrm{e}}}$:

$$u'(c(t,\omega_t)) \approx u'(\overline{c_{\mathrm{e}}}) + u''(\overline{c_{\mathrm{e}}})(c(t,\omega_t) - \overline{c_{\mathrm{e}}}) + \frac{1}{2}u'''(\overline{c_{\mathrm{e}}})(c(t,\omega_t) - \overline{c_{\mathrm{e}}})^2$$

Since $E[(c(t,\omega_t)|\theta(t,\omega_t) = 1] = \overline{c_{\mathrm{e}}}$ by definition, it follows that

$$\begin{aligned} Eu'(c_{\mathrm{e}}) &= E[u'(c(t,\omega_t))|\theta(t,\omega_t) = 1] \\ &= u'(\overline{c_{\mathrm{e}}}) + \frac{1}{2}u'''(\overline{c_{\mathrm{e}}})E\left[(c(t,\omega_t) - \overline{c_{\mathrm{e}}})^2|\theta(t,\omega_t) = 1\right] \end{aligned}$$

and substituting in the definitions of ρ and γ yields (16). If $u'''=0$, $\rho=0$, and (16) reduces to $u'(\overline{c_{\mathrm{e}}})$. Similar reasoning establishes the result for the unemployed case. □

When utility is well approximated by a quadratic function in the region of consumption fluctuations within an employment state, only the average consumption level is needed to determine average marginal utility. This is a standard certainty equivalence result for quadratic functions. If one wishes to take third order terms into account, the formula also depends on the coefficient of relative prudence ρ and the coefficient of variation of consumption in each state.

3.5. Welfare gain from UI

I now derive an expression for the welfare gain from an increase in b relative to the welfare gain of a permanent one-dollar increase in consumption in the employed state $\left(\frac{\mathrm{d}V/\mathrm{d}b}{(1-D)Eu'(c_{\mathrm{e}})}\right)$. This expression provides a simple money-metric to compute the welfare gain associated with social insurance.

Lemma 3. *The change in welfare from an increase in b relative to the change in welfare from a permanent increase in consumption while employed is approximately*

$$\frac{\mathrm{d}V/\mathrm{d}b}{(1-D)Eu'(c_{\mathrm{e}})} \approx \frac{D}{1-D}\left[\frac{\Delta c}{c}(b)\gamma\left[1+\frac{1}{2}\rho\frac{\Delta\bar{c}}{c}(b)\right] - \frac{\varepsilon_{D,b}}{1-D}\right] \tag{17}$$

where $\frac{\Delta\bar{c}}{c} = \frac{\overline{c_{\mathrm{e}}}-\overline{c_{\mathrm{u}}}}{\overline{c_{\mathrm{e}}}}$=mean consumption drop due to unemployment; $\gamma = -\frac{u''(\overline{c_{\mathrm{e}}})}{u'(\overline{c_{\mathrm{e}}})}\overline{c_{\mathrm{e}}}$=relative risk aversion; $\rho = -\frac{u'''(\overline{c_{\mathrm{e}}})}{u''(\overline{c_{\mathrm{e}}})}\overline{c_{\mathrm{e}}}$=relative prudence; $\varepsilon_{D,b} = \frac{\mathrm{dlog}D(c,x)}{\mathrm{dlog}b}$=elasticity of duration w.r.t. benefits.

Proof . From Lemma 1, we have

$$\frac{\mathrm{d}V}{\mathrm{d}b} = -\frac{\mathrm{d}\tau}{\mathrm{d}b}(1-D)Eu'(c_{\mathrm{e}}) + DEu'(c_{\mathrm{u}}) \tag{18}$$

Differentiating the UI budget constraint implies

$$\frac{\mathrm{d}\tau}{\mathrm{d}b} = \frac{D(1-D) + b\frac{\mathrm{d}D}{\mathrm{d}b}}{(1-D)^2}$$

and plugging this expression into (18) and simplifying gives:

$$\frac{\mathrm{d}V}{\mathrm{d}b} = DEu'(c_\mathrm{u}) - DEu'(c_\mathrm{e})\left[1 + \frac{\varepsilon_{D,b}}{1-D}\right] \tag{19}$$

Rearranging (19), it follows that

$$\frac{\mathrm{d}V/\mathrm{d}b}{(1-D)Eu'(c_\mathrm{e})} = \frac{D}{1-D}\left\{\frac{Eu'(c_\mathrm{u}) - Eu'(c_\mathrm{e})}{Eu'(c_\mathrm{e})} - \frac{\varepsilon_{D,b}}{1-D}\right\} \tag{20}$$

To simplify this expression, apply the quadratic approximation given in (15) of Lemma 2 for $Eu'(c_\mathrm{u})$ and $Eu'(c_\mathrm{e})$ to obtain

$$\frac{\mathrm{d}V/\mathrm{d}b}{(1-D)Eu'(c_\mathrm{e})} = \frac{D}{1-D}\left\{\frac{u'(\overline{c_\mathrm{u}}) - u'(\overline{c_\mathrm{e}})}{u'(\overline{c_\mathrm{e}})} - \frac{\varepsilon_{D,b}}{1-D}\right\}$$

The first term in this expression can be approximated using a Taylor expansion analogous to (9) in Proposition 1. Using the definitions of γ, ρ, and $\frac{\Delta\bar{c}}{c}$ yields (17). □

Lemma 3 shows that the three reduced-form parameters identified by Baily, along with the correction factor ρ, are sufficient to determine the welfare gains from social insurance in a general setting. The result indicates that the welfare gains from social insurance are greater when shocks are more common ($\frac{D}{1-D}$ large). It also confirms the intuition that larger consumption-smoothing benefits and a smaller duration response yield a larger welfare gain.

The key equation in the analysis of the general case is (20), which gives an *exact* expression for the marginal welfare gain of increasing b in terms of expected marginal utilities and the duration elasticity. Lemma 3 proceeds to simplify this equation using the quadratic approximation given in Lemma 2 for $Eu'(c_\mathrm{e})$ and $Eu'(c_\mathrm{u})$ instead of the cubic approximation given in (16).[7] This is because the quadratic approximation for these terms is reasonably accurate for the purpose of computing $\frac{\mathrm{d}V/\mathrm{d}b}{(1-D)Eu'(c_\mathrm{e})}$ and $b*$. If the third-order approximations were used to simplify (20) instead, we would obtain

$$\frac{\mathrm{d}V/\mathrm{d}b}{(1-D)Eu'(c_\mathrm{e})} = \frac{D}{1-D}\left\{\left[\frac{\Delta\bar{c}}{c}(b*)\gamma\left[1 + \frac{1}{2}\rho\frac{\Delta\bar{c}}{c}(b*)\right] + 1\right]F - 1 - \frac{\varepsilon_{D,b}}{1-D}\right\} \tag{21}$$

where $F = \frac{1+\gamma\rho s_\mathrm{u}^2}{1+\gamma\rho s_\mathrm{e}^2}$ is a correction factor that accounts for differences in the volatility of consumption in the two states. This equation shows that the bias of the quadratic approximation is proportional to the *ratio* of the coefficient of variation of consumption in the unemployed and employed states. A rough estimate from panel data on consumption in the Consumer Expenditure Survey suggests that s_u and s_e are around 20%, with $\frac{s_\mathrm{u}}{s_\mathrm{e}}$ between $\frac{1}{2}$ and 2. In this range, using a power utility function and other parameters chosen as described in the earlier calibration exercise, the exact value of $\frac{\mathrm{d}V/\mathrm{d}b}{(1-D)Eu'(c_\mathrm{e})}$ and the approximate value given by (17) differ by less than 2%. I therefore proceed by assuming $Eu'(c_\mathrm{e}) \approx u'(\overline{c_\mathrm{e}})$ and $Eu'(c_\mathrm{u}) \approx u'(\overline{c_\mathrm{u}})$ below.

3.6. Optimal benefit level

The generalized formula for the optimal benefit level follows directly from the preceding result on welfare gains.

[7] To be clear, note that Lemma 3 still uses a third-order approximation for $u'(\overline{c_\mathrm{u}}) - u'(\overline{c_\mathrm{e}})$ as in the static model; it is only when approximating $Eu'(c_e)$ and $Eu'(c_\mathrm{u})$ that we are ignoring the u''' terms.

Proposition 2. *The optimal benefit rate b^* is approximately defined by*

$$\frac{\Delta\bar{c}}{c}(b^*)\gamma\left[1+\frac{1}{2}\rho\frac{\Delta\bar{c}}{c}(b^*)\right]\approx\frac{\varepsilon_{D,b}}{1-D} \tag{22}$$

where $\frac{\Delta\bar{c}}{c}\gamma$, ρ, *and* $\varepsilon_{D,b}$ *are defined as in Lemma 3.*

Proof.

(a) Necessity. The optimal benefit rate must satisfy

$$\mathrm{d}V/\mathrm{d}b(b^*)=0 \tag{23}$$

Using the expression for $\mathrm{d}V/\mathrm{d}b$ in (17) implies

$$\frac{\Delta c}{c}(b^*)\gamma\left[1+\frac{1}{2}\rho\frac{\Delta\bar{c}}{c}(b^*)\right]-\frac{\varepsilon_{D,b}}{1-D}\approx 0$$

and rearranging yields (22).

(b) Sufficiency. To establish that the b^* defined by (22) is a global maximum, we show that $V(b)$ is strictly concave in b. Differentiating the expression for $\mathrm{d}V/\mathrm{d}b$ in (19) gives $\mathrm{d}^2V/\mathrm{d}b^2<0$ under the conditions of Assumption 4, completing the proof. □

The formula for b^* in the general case (22) coincides with the corresponding condition (2) for the static model, with two exceptions. First, the inputs reflect *average* behavioral responses across states and time. The consumption drop that is relevant is the percentage difference between average consumption while employed and unemployed.[8] The $\varepsilon_{d,b}$ term is the effect of a 1% increase in b on the fraction of his life the agent spends unemployed. This is equivalent to the effect of an increase in b on the average unemployment duration if the frequency of layoffs is not affected by b. If benefits affect the frequency of layoffs, one must take both the average duration effect and the layoff elasticity into account to compute $\varepsilon_{d,b}$. The second difference in the formula for the general case is that it has an added $\frac{1}{1-d}$ term that magnifies the elasticity of durations with respect to benefits. This is because raising consumption while unemployed by \$1 generates not only the added cost of providing benefits during a longer duration, also causes a reduction in tax collection because the agent spends less time employed. In practice, the latter effect is likely to be small, especially if the agent is usually employed so that $1-d$ is close to 1.

It is interesting to note the connection between Proposition 2 and results from the literature on optimal commodity taxation. The optimal social insurance problem analyzed above is formally equivalent to the choice of optimal commodity taxes in a second-best environment, where the commodities correspond to state-contingent consumption. Ramsey's (1927) classic analysis of commodity taxation shows that the formula for optimal tax rates does not depend on untaxed behavioral responses in a first-best setting. Similar results on the irrelevance of untaxed choice variables and constraints on utility maximization can be obtained in a second-best environment

[8] Extending this logic, heterogeneity across agents in behavioral responses (as documented in Crossley and Low, 2005) does not affect the formula for b^* in a universal-benefit program if one uses population averages for $\frac{\Delta c}{c}$ and $\varepsilon_{d,b}$ in (22). If there is also heterogeneity in γ across individuals, aggregation of utilities is more complicated and depends on the structure of the social welfare function.

(see Green, 1961 or Auerbach, 1985). Analogously, the present paper shows that the formula for the optimal benefit rate in a social insurance problem does not depend on the agent's choice variables and constraints.

3.7. Implementing the formula

The formula in (22) is deceptively simple, in the sense that empirical estimation of its key inputs requires careful consideration of several issues. First, it is important to recognize that the parameters $\varepsilon_{D,b}$ and $\frac{\Delta\bar{c}}{c}(b)$ involve *total* (not partial) derivatives of $D(c,x)$ and $\frac{\Delta\bar{c}}{c}$ with respect to b. For example, to compute $\varepsilon_{D,b}$, one must account for the fact that higher b will change many behaviors (such as savings rates), and each of these behavioral responses will feed back into the choice of D. Estimating the total response would therefore be difficult if one were to try to identify all the feedback effects separately. However, reduced-form studies that compare unemployment durations across states/times that differ only in UI benefit levels (and UI tax rates) identify the total derivative of interest, because all other behaviors are allowed to vary endogenously.

A second issue in implementing (22) is that one must estimate the effect of UI benefits on the average consumption drop over a lifetime. Existing empirical estimates of consumptionsmoothing benefits in the literature, such as Gruber (1997), depart from this ideal in two respects: (1) they analyze single spells within a lifetime and (2) they only estimate the change in consumption from the period immediately before (or after) unemployment to the unemployment spell. The use of data only on individual spells is not a serious concern to the extent that the cross-sectional distribution of the individuals in a given sample is representative of average behavior for a given individual over his lifetime. However, the focus on only high-frequency consumption changes around job loss could be more problematic, e.g. if consumption trends upward or downward over time while agents are employed. An useful direction for future empirical work would be to estimate longer-run consumption-smoothing elasticities.

A third concern in (22) is that one must estimate the parameters in the context that they are applied. For example, recent studies have found that risk-aversion can vary significantly across the scale of shocks. Risk aversion (γ) with respect to small, temporary shocks such as unemployment may be much greater than risk aversion with respect to large shocks such as disability because of rigidities such as consumption commitments (Chetty and Szeidl, 2006). Similarly, risk aversion may differ sharply across income levels or economies. Hence, in implementing (22), one must be careful to use estimates of γ and other parameters that are appropriate for the context of interest.

3.8. Implications

Proposition 2 implies that many of the extensions that have followed Baily's analysis do not require a reformulation of the optimal benefit rule proposed here, because the four key parameters in (22) remain sufficient statistics for the purpose of computing b^*. Some notable examples include:

1. Borrowing constraints. If the consumption-smoothing benefits of UI are estimated using data on consumption rather than simulated based on assumptions about primitives, the particular features of the underlying borrowing constraints that agents face become irrelevant. Tighter borrowing constraints (or low levels of savings) will generate a larger observed consumption

drop in the data, and therefore raise the optimal benefit level, consistent with the results of Flemming (1978) and Crossley and Low (2005).

2. Private insurance markets. Eq. (22) remains valid when agents can make private insurance arrangements because these are simply additional choice variables in the general case. Intuitively, the possibility that social insurance may crowd out private insurance is captured through the $\frac{\Delta c}{c}$ parameter, which will be smaller (and therefore imply a lower b^*) if agents have already made informal or formal insurance arrangements. One restrictive aspect of the formula, however, is that the private insurance contracts cannot involve any moral hazard. Moral hazard in private contracts would create an additional wedge between the private and social marginal costs of search, violating the "no externalities" assumption maintained throughout the analysis. Insurance arrangements such as spousal labor supply may not involve moral hazard because the household internalizes the costs of insurance. However, insurance contracts purchased through a market are likely to involve moral hazard. In ongoing work, we are exploring whether reduced-form expressions for the optimal benefit level can be obtained when both social and private insurance involve moral hazard.
3. Multiple consumption goods. The proposition shows that it is sufficient to obtain consumption-smoothing estimates for a single good (e.g., food), provided that the appropriate risk aversion parameter (e.g., curvature of utility over food) is used in conjunction with this estimate. This is because all the other consumption goods can be placed in the set of x other choice variables. This point is relevant for two reasons. First, one may be concerned that existing estimates of consumption-smoothing have limited applicability because they only consider a few categories of consumption such as food (Gruber, 1998). The result here suggests that from a normative perspective, it is not critical to have consumption-smoothing estimates for the full consumption bundle. Second, there is a concern that the durability of consumption may affect optimal UI policy. Browning and Crossley (2003) show that postponing expenditures on small durables such as clothes can provide households an additional smoothing channel via an "internal capital market," thereby lowering the optimal level of unemployment insurance. These effects can be captured through additional consumption goods and constraints in the general case analyzed above, and ultimately do not affect b^* conditional on the consumption-smoothing elasticity for food.
4. Search and human capital benefits of UI. Unemployment benefits could affect subsequent wages by subsidizing search and improving job match quality. UI could also increase incentives for risk-averse workers to undertake risky human capital investments (Brown and Kaufold, 1988). Under the assumption that UI is financed by a lump-sum tax, the increment in wages from these effects has no effect on UI tax collections, and is therefore fully internalized by the worker. Consequently, these effects can be ignored in calculating the optimal level of benefits; only the consumption-smoothing benefits need to be considered.
5. Leisure value of unemployment. Leisure is also simply another choice variable in the general framework, and thus has no impact on the optimal benefit equation. The intuition that all else held fixed, greater leisure value should raise b^* comes through the $\frac{\Delta c}{c}$ term. If unemployment has higher leisure value (or if there are search benefits), the agent is willing to sacrifice more consumption to take more time off, leading to a larger consumption drop and higher optimal benefit rate. However, *conditional* on knowing $\frac{\Delta c}{c}$ and $\varepsilon_{D,b}$, leisure or search benefits have no additional effect on the optimal benefit rate because they are already taken into account via agent optimization.
6. Dynamic search and savings behavior. Lentz (2004) and others have structurally estimated job search models which permit agents to optimize savings dynamically and allow for rich

search dynamics. These models are considerably more complex than the static Baily framework, but are nested within the general case considered here. Hence, they should not change conclusions about the optimal benefit rate if it is calculated using (22).

The robustness of (22) to variations in the underlying model suggests that it should provide a reliable estimate of the optimal level of social insurance. Unlike the alternative "structural" approach, there is no need to explicitly specify the agent's discount factor, the functional form of u, the stochastic process for θ as a function of search effort, etc. As Gruber (1997) observes, each of these parameters is difficult to estimate, making it difficult to implement the structural approach credibly. However, the reduced-form approach also comes with some potential dangers that arise from failing to specify the underlying structure. The next section describes these concerns.

4. The apparent irrelevance of some parameters

A surprising feature of the optimal benefit rate formula (22) is that it does *not* depend on many elasticities that one would think should affect the costs and benefits of unemployment insurance. For example, prior studies have investigated the effects of UI benefits on reemployment wages (Ehrenberg and Oaxaca, 1976), reservation wages (Feldstein and Poterba, 1984), pre-unemployment savings (Engen and Gruber, 2001), spousal labor supply (Cullen and Gruber, 2000), and job-match quality (Centeno, 2004). According to the formula, none of these empirical results is relevant to the normative analysis of unemployment insurance.

How can the formula be reconciled with the intuition that these other factors should matter for b^*? The key is to recall that the elasticities that enter the formula are all functions of other aspects of the agent's behavior and preferences. The effects described in the previous paragraph affect b^* by altering the values of the main inputs (γ, ρ, $\frac{\Delta c}{c}$, and $\varepsilon_{D,b}$) that enter the formula directly. The formula for b^* could alternatively be written as a function of these auxiliary parameters. I now illustrate this point formally by focusing on a specific example–the importance of income vs. substitution effects in determining the optimal benefit level–where the potential pitfalls in applying (22) are apparent.

4.1. Income and substitution effects

A central insight of the literature on optimal taxation is that the efficiency consequences of taxation, and hence optimal tax rates, are determined by substitution elasticities (and not uncompensated elasticities). Since a social insurance program is abstractly a particular type of redistributive tax, it may be surprising that the optimal benefit rate appears to depend on the total uncompensated elasticity of unemployment durations with respect to benefits and not just the substitution elasticity. The reason for the apparent discrepancy between the two intuitions is that formulas for optimal taxes or benefits have multiple representations. In the tax literature, a Slutsky decomposition has proved useful in interpretation of the results, so formulas are typically written in terms of substitution and income effects. However, one could instead write formulas for optimal tax rates in terms of total uncompensated elasticities (Sandmo, 1976). Similarly, one can obtain an alternative representation for the optimal UI benefit level in terms of income and substitution elasticities using a Slutsky decomposition.

Exploring the implications of such an alternative representation is particularly interesting because there is accumulating evidence indicating that unemployment and UI benefits have substantial income effects. Mincer (1962) found that married women's labor supply responds 2–3 times as much to transitory fluctuations in husbands' incomes due to unemployment as it does to permanent differences in husbands' incomes. Cullen and Gruber (2000) exploit variation in UI benefit levels to estimate an income elasticity for wives' labor supply between −0.49 and −1.07. More recently, Chetty (2005) finds that lump-sum severance payments, which have pure income effects, significantly raise durations. If income effects are large relative to substitution effects, one would intuitively expect that b^* should be higher, but it is not obvious how this would occur if one computes b^* using (22).

To derive a formula for b^* in terms of income and substitution effects, let us return to the static model of Section 2.1 for simplicity. Suppose agents receive a lump sum severance payment of b_0 upon unemployment. The first order condition that determines the agent's choice of D in the unemployed state is then

$$(w-b)u_c(c_u)=\psi_D(D) \tag{24}$$

where $c_u = A_0 + b_0 + bD + w(1-D)$ is consumption in the unemployed state. Intuitively, the agent equates the marginal benefit of extending his duration by 1 day, ψ_D, with the marginal consumption cost of doing so, which is the foregone wage $(w-b)$ times the marginal utility of consumption.

Now consider the comparative statics implied by this first order condition. Implicit differentiation of (24) with respect to b_0 and b yields

$$\frac{\partial D}{\partial b_0}=\frac{(w-b)u_{cc}}{(w-b)^2u_{cc}+\psi_{DD}}$$

$$\frac{\partial D}{\partial b}=\frac{D(w-b)u_{cc}-u_c}{(w-b)^2u_{cc}+\psi_{DD}} \tag{25}$$

Using a Slutsky decomposition, the substitution effect $\frac{\partial D^c}{\partial b}$ for duration (which equals one minus labor supply here) is given by

$$\frac{\partial D^c}{\partial b}=\frac{\partial D}{\partial b}-D\frac{\partial D}{\partial b_0} \tag{26}$$

This implies

$$\frac{\partial D/\partial b_0}{\partial D/\partial b-D\partial D/\partial b_0}=\frac{\partial D/\partial b_0}{\partial D^c/\partial b}=-(w-b)\frac{u_{cc}}{u_c}$$

$$\Rightarrow \gamma=-\frac{u_{cc}}{u_c}c_u=\frac{\partial D/\partial b_0}{\partial D^c/\partial b}\frac{c_u}{w-b} \tag{27}$$

Eq. (27) shows that γ is related to the ratio of the income and substitution effects of UI benefits on unemployment durations. This connection between risk aversion and duration elasticities is a special case of Chetty's (2006) result that labor supply elasticities place bounds on risk aversion in an expected utility model with arbitrary non-separable utility. To see the rough intuition for this result, consider the effects of lump-sum and proportional benefit reductions on duration. An agent's duration response to a proportional benefit (b) reduction is directly related to u_c, the marginal utility of consumption: the larger is u_c, the greater the benefit of an additional dollar of income, and the more the agent will work when his effective wage $(w-b)$ goes up. The duration response to an increase in the severance payment (b_0) is related to how much the marginal utility of consumption changes as consumption changes, u_{cc}. If u_{cc} is large, the marginal utility of consumption rises sharply as income falls, so the agent will shorten his duration a lot to earn more money when his severance pay falls. Since γ is proportional to u_{cc}/u_c, it follows that there is a connection between γ and the ratio of income and price elasticities of benefits.[9]

Eq. (27) implies that large income effects do indeed generate a higher b^*, by raising the risk aversion parameter. Yet, conditional on the value of γ, $\partial d/\partial b_0$ and $\partial d^c/\partial b$ play no role in determining b^*. This observation illustrates why the reduced-form formula should be used cautiously. When (22) is calibrated with a low value of γ – as in some of the cases considered by Baily (1978) and Gruber (1997) – one is at risk of contradicting the evidence of large income effects described above. Put differently, (22) is only one representation of the formula for optimal benefits. Another representation would involve income and substitution elasticities and the consumption drop, but not γ. Since this alternative representation might yield different conclusions about b^*, it is important to check whether the inputs used to calculate b^* are consistent with other estimates of behavioral responses.[10]

The income effect analysis above is just one example of the danger in applying the reduced-form formula without carefully considering the restrictions implied by a fully specified structural model. The broader point is that while only a small set of parameters need to be estimated to draw normative conclusions about social insurance, estimates of other elasticities can be valuable in performing "overidentification" tests of the validity of the primary inputs.

5. Conclusion

This paper has shown that a simple, empirically implementable formula can be used to compute the welfare gains and optimal level of social insurance in a wide class of stochastic dynamic models. Although the analysis focused on unemployment, this formula can also be applied to analyze other policies (such as disability insurance or welfare programs) if one restricts attention to the optimal policy in a two-state model with constant benefits in one state and a constant tax in the other. Hence, reduced-form empirical estimates of behavioral responses can be used to obtain robust estimates of the optimal size of many large government expenditure programs.

While the formula derived here offers an improvement over prior studies, there are many limitations to the analysis. Some interesting possibilities for further work include:

1. Time-varying benefits. This paper assumed that unemployment benefits are offered at a constant level indefinitely. Simulation results in Davidson and Woodbury (1997) indicate that the optimal benefit rate can differ significantly if benefits are offered only for a finite

[9] The derivation here assumes that there is no complementarity between labor and consumption. More generally, γ is a function of the ratio of income and price elasticities as well as the degree of complementarity. See Chetty (2006) for details.

[10] This point applies equally to the optimal tax literature. There, optimal tax rates depend on income and substitution elasticities, but could equivalently be written in terms of γ instead. One could test whether the different representations yield similar predictions for optimal taxation.

duration, as in the US. A useful direction for future work would be to derive a reduced-form formula for the optimal benefit rate when benefits are offered for a fixed length of time. It would also be interesting to analyze whether the optimal duration of benefits (or the path of benefits) can be computed using general formulas that are robust to modelling details.

2. Fiscal externalities. I assumed that the only distortionary tax/subsidy in the economy is the UI benefit. In practice, many behaviors are taxed, and these taxes create "fiscal externalities" that could change the formula for the optimal benefit rate. For example, higher UI benefits will in general lower private savings, which could in turn reduce tax collections from capital gains or dividends. This tax revenue effect is not internalized by the agent and therefore affects the optimal benefit rate directly. It would be useful to determine the magnitude of such fiscal externalities too assess whether they affect the calculation of the optimal benefit rate significantly.
3. General equilibrium effects. In the models analyzed here, all behavioral responses to UI were solely determined by the agent. This assumption was important because the envelope conditions used to obtain the formula for optimal benefits relied on the idea that all endogenous variables in the model are chosen to maximize the agent's utility. Obtaining a reduced-form formula that takes equilibrium responses by firms into account would be useful.
4. Endogenous take-up. The analysis assumed that all agents receive benefits upon unemployment automatically. In practice, take-up rates for social insurance programs are far below 100% and are sensitive to the level of benefits (Anderson and Meyer, 1997). Allowing for endogenous take-up may therefore have quantitatively significant impacts on the optimal benefit level.[11]
5. Myopic agents. The envelope arguments above rely on the assumption that agents are optimizing. If agents experience large consumption drops during unemployment because they are myopic and do not save enough, the formula for the optimal benefit level may be different.

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[11] See Davidson and Woodbury (1997) for simulation results regarding the effect of endogenous take-up on the optimal benefit rate.

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