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THE SIMPLE ECONOMICS OF SALIENCE AND TAXATION

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ABSTRACT

This paper derives empirically implementable formulas for the incidence and efficiency costs of taxation that account for tax salience effects as well as other optimization errors. Contrary to conventional wisdom, the formulas imply that the economic incidence of a tax depends on its statutory incidence and that a tax can create deadweight loss even if it induces no change in demand. The results are derived using simple supply and demand diagrams and familiar notions of consumer and producer surplus. The approach to welfare analysis proposed here yields robust formulas because it does not require specification of a positive theory for why agents fail to optimize with respect to tax policies.

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A central assumption in public economics is that agents optimize fully with respect to tax policies. For example, Frank P. Ramsey's (1927) seminal analysis of optimal commodity taxation assumes that agents respond to tax changes in the same way as price changes. Canonical results on tax incidence, efficiency costs, and optimal income taxation (e.g. Arnold C. Harberger 1964, James A. Mirrlees 1971, Anthony B. Atkinson and Joseph E. Stiglitz 1976) all rely on full optimization with respect to taxes.

Contrary to the full optimization assumption, there is accumulating evidence which suggests that individuals optimize imperfectly with respect to many types of tax and transfer policies. For example, Raj Chetty, Adam Looney, and Kory Kroft (2007) analyze the effect of "salience" on behavioral responses to commodity taxation. They find that commodity taxes that are included in the posted prices that consumers see when shopping (and are thus more salient) have much larger effects on demand.[1] Kelly Gallagher and Erich Muehlegger (2008) show that more salient sales tax waivers given at the time of purchase have seven times as large an effect on hybrid vehicle purchases as income tax credits of an equivalent amount. Chetty and Emmanuel Saez (2009) show using a field experiment that providing simple information about the marginal incentive structure of the Earned Income Tax Credit leads to significant changes in subsequent labor supply and earnings behavior. In Xavier Gabaix and David I. Laibson's (2006) terminology, these studies show that many tax policies are "shrouded attributes." Such inattention and imperfect optimization may be prevalent in the case of taxation because tax systems are complex and nontransparent. Income tax schedules are highly nonlinear, benefit-tax linkages for social insurance programs are opaque (e.g. social security taxes and benefits), and taxes on commodities are often not displayed in posted prices (sales taxes, hotel city taxes, vehicle excise fees).

Motivated by this empirical evidence, this paper analyzes the implications of salience effects and other optimization errors for the welfare consequences of tax policy. The challenge

[1] I use "tax salience" to refer to the visibility of the tax inclusive price. When taxes are included in the posted price, the total tax-inclusive price is more visible but the tax rate itself may be less clear. There is a longstanding theoretical literature on "fiscal illusion" which discusses how the lack of visibility of tax rates may affect voting behavior and the size of government (John S. Mill 1848). Unlike that literature, I define salience in terms of the visibility of the tax inclusive price because I focus on behaviors that optimally depend on total tax inclusive prices rather than behaviors which depend on the tax rate itself.

in this analysis – as in behavioral public economics more generally – is the calculation of welfare when behavior is inconsistent with full optimization (B. Douglas Bernheim and Antonio Rangel 2009, Jerry R. Green and Daniel Hojman 2009). One approach to this problem is to specify a positive model for why agents deviate from full optimization and analyze welfare costs within that model. This is the approach taken by Chetty, Looney, and Kroft (2007), who derive formulas for the incidence and efficiency costs of taxes in a bounded rationality model of tax salience. Although useful in obtaining some insights into welfare implications, this approach has the shortcoming of relying on particular assumptions about what drives deviations from full optimization. Bounded rationality is not the only model of inattention; models of forgetfulness or cue theories of attention could also generate salience effects, and could potentially lead to different welfare implications.

In this paper, I develop an alternative method of characterizing the welfare consequences of taxation when agents optimize imperfectly that does not rely on a specific positive model of behavior. The approach rests upon two general assumptions: (1) tax policies affect welfare only through their effects on the consumption bundle chosen by the agent and (2) consumption choices when prices are fully salient – e.g., when there are no taxes – are consistent with full optimization. Under these assumptions, I derive formulas for the incidence and efficiency costs of taxation that depend only on the empirically observed demand function and not on the underlying model which generates that demand function. Intuitively, there are two demand curves that together are sufficient statistics for welfare calculations when individuals make optimization errors: the *tax-demand* curve, which tells us how demand varies with taxes that are not included in posted prices, and the *price-demand* curve, which tells us how demand varies as (fully salient) posted prices change. I use the tax-demand curve to determine the effect of the tax on behavior and then use the price-demand curve to calculate the effect of that change in behavior on welfare. The price-demand curve can be used to recover the agent's underlying preferences and calculate welfare because it is generated by optimizing behavior.

The benefits of this approach to welfare analysis are its simplicity and adaptability. The formulas for excess burden and incidence can be derived using supply and demand diagrams and familiar notions of consumer and producer surplus. The formulas differ from the standard Harberger (1964) expressions by a single factor – the ratio of the compensated tax elasticity to the compensated price elasticity. Thus, one can calculate the (partial equilibrium) deadweight cost and incidence of any tax policy by estimating both the tax and price elasticities instead of just the tax elasticity as in the existing empirical literature. Although the welfare analysis is motivated by evidence of salience effects, the formulas account for *all* errors that consumers may make when optimizing with respect to taxes.[2] For example, confusion between average and marginal income tax rates (Charles de Bartolome 1995, Jeffrey B. Liebman and Richard J. Zeckhauser 2004, Naomi E. Feldman and Peter Katuščák 2006) or over perception of estate tax rates (Robert J. Blendon, *et al.* 2003, Joel B. Slemrod 2006) can be handled using exactly the same formulas, without requiring knowledge of individuals' tax perceptions and information set.[3]

In addition to providing quantitative guidance about welfare consequences, the formulas derived here challenge widely held qualitative intuitions based on the full optimization model. First, the agent who bears the statutory incidence of a tax bears more of the economic incidence, violating the classic tax neutrality result in competitive markets. Second, a tax increase on a normal good can have a substantial efficiency cost even when demand for the good does not change by distorting budget allocations. Finally, holding fixed the tax elasticity of demand, an increase in the price elasticity of demand *reduces* deadweight loss and *increases* incidence on consumers.

The approach to welfare analysis in this paper can be viewed as an application of Bernheim and Rangel's (2009) choice-based approach, in which the choices when taxes are salient reveal an agent's true rankings. It is also an example of the recent sufficient statistic approach in public economics, in which welfare implications are derived from high-level elasticities rather than structural primitives (Chetty 2009).

The remainder of the paper is organized as follows. Section I sets up a simple model

[2] The formulas do not, however, permit errors in optimization relative to salient prices. Such errors can be accommodated by isolating a condition where the true price elasticity is revealed and applying the formulas here.

[3] Liebman and Zeckhauser (2004) analyze optimal income taxation in a model where individuals misperceive tax schedules because of "ironing" or "spotlighting" behavior. The approach proposed in the present paper does not require assumptions about whether individuals iron, spotlight, or respond in some other way to the tax schedule, as any of these behaviors are captured in the empirically observed tax and wage elasticities of labor supply.

of demand with salience effects. Section II characterizes tax incidence in this model. Section III characterizes efficiency costs, which is a more complex problem because additional assumptions are required to calculate welfare changes when agents optimize imperfectly. Section IV concludes.

I Setup

Consider an economy with two goods, x and y. The government levies two specific (unit) taxes on good x: an "excise" tax t^E that is included in the posted price and a "sales" tax t^S that is not included in the posted price.[4] The only distinction between the two taxes is their salience: the excise tax is perfectly salient because the excise-tax-inclusive price is visible, whereas the sales tax is not fully salient. I use the excise and sales tax terminology to match commodity taxes, but the formulas below can be applied to any tax, including labor and capital income taxes.

Let $t = t^E + t^S$ denote the total tax on good x. Good y, the numeraire, is untaxed. Let p denote the pretax price of x and $q = p + t$ denote the tax-inclusive price of x. As is standard in partial equilibrium analyses, assume that the tax revenue is not spent on the taxed good (i.e. it is used to buy y or thrown away).[5] The tools developed below can be adapted to analyze Pigouvian taxes intended to correct behavior, but I defer that analysis to future work.

Consumption. The representative consumer has wealth Z and has utility $u(x) + v(y)$. In the benchmark full-optimization model, the agent chooses a consumption bundle $(x^*(p + t^E, t^S, Z), y^*(p + t^E, t^S, Z))$ that satisfies

$$\begin{aligned} u'(x^*) &= (p+t)v'(y^*) \\ (p+t)x^* + y^* &= Z \end{aligned}$$

[4] I analyze specific rather than ad valorem (percentage of price) taxes to simplify the algebra. The incidence and excess burden of introducing an ad valorem tax τ^S when there are no pre-existing taxes can be calculated by replacing t^S by τ^S and $\frac{\partial x}{\partial t^S}$ by $\frac{\partial x}{\partial \tau^S}$ in the derivative-based formulas in Propositions 1-3.

[5] The welfare analysis focuses solely on the costs of raising tax revenue, taking the benefits of a given amount of revenue as invariant to the tax system used to generate it. For example, I ignore the possibility that more visible taxes may constrain inefficient spending by politicians (Amy N. Finkelstein 2007).

This model implies $\frac{\partial x^*}{\partial p} = \frac{\partial x^*}{\partial t}$, contradicting the empirical evidence described in the introduction. To allow for different responses to prices and taxes, let $x(p + t^E, t^S, Z)$ denote the empirically observed demand for x as a function of the posted price, sales tax, and wealth and $y(p+t^E, t^S, Z)$ the corresponding demand function for y. I do not place structure on the positive model that generates $(x(p + t^E, t^S, Z), y(p + t^E, t^S, Z))$ other than to assume that the demand functions are smooth and that the choices are feasible:

$$(p + t)x(p + t^E, t^S, Z) + y(p + t^E, t^S, Z) = Z$$

Define the degree of under reaction to the tax as

$$\theta = \frac{\partial x(p + t^E, t^S, Z)}{\partial t^S} / \frac{\partial x(p + t^E, t^S, Z)}{\partial p} = \frac{\varepsilon_{x,q|t^S}}{\varepsilon_{x,q|p}}$$

where $\varepsilon_{x,q|t^S} = -\frac{\partial x}{\partial t^S} \frac{q}{x(p+t^E, t^S, Z)}$ measures the percentage change in demand caused by a 1 percent increase in the total price of good x through a tax change, while $\varepsilon_{x,q|p} = -\frac{\partial x}{\partial p} \frac{q}{x(p+t^E, t^S, Z)}$ represents the analogous measure for a 1 percent increase in q through a change in p. When discussing the intuition for the results below, I will focus on the case where $\theta < 1$ and interpret θ as a measure of the degree of inattention to the tax. However, the analysis permits $\theta > 1$ and more generally permits $\frac{\partial x}{\partial t}$ to differ from $\frac{\partial x}{\partial p}$ for any reason, not just inattention.[6] The formulas derived below therefore account for any errors that consumers may make when optimizing with respect to taxes.

Production. Assume that the supply of the numeraire good y is perfectly elastic. This assumption shuts down general equilibrium effects by ensuring that the price of y is unaffected by the tax on x. Good x is produced by price-taking firms, which use $c(S)$ units of y to produce S units of x. The marginal cost of production is weakly increasing: $c'(S) > 0$ and $c''(S) \geq 0$. Let $\pi(S) = pS - c(S)$ denote the representative firm's profits at a given pretax price p and level of supply S. Assuming that firms optimize perfectly, the supply function for good x is implicitly defined by the first-order condition for S in the profit-maximization

[6] Although the empirical studies described above find that $\theta < 1$, this need not be the case for all taxes. The opaque estate tax system, for example, appears to cause many individuals to over-perceive tax rates on wealth (Slemrod 2006).

problem: $p = c'(S(p))$.[7] Let $\varepsilon_{S,p} = \frac{\partial S}{\partial p}\frac{p}{S(p)}$ denote the price elasticity of supply.

II Incidence

How is the burden of a tax shared between consumers and producers in competitive equilibrium when consumers optimize imperfectly with respect to taxes? I derive formulas for the incidence of the sales tax on producers and consumers which parallel the derivations of Laurence J. Kotlikoff and Lawrence H. Summers (1987) for the full-optimization case. As is standard in the literature on tax incidence, I use $D(p, t^S, Z)$ instead of $x(p, t^S, Z)$ to refer to the demand curve in this subsection. Let $p = p(t^E, t^S)$ denote the equilibrium pretax price that clears the market for good x as a function of the tax rates. The market clearing price p satisfies

$$D(p + t^E, t^S, Z) = S(p) \tag{1}$$

Implicit differentiation of (1) yields the following results.

Proposition 1 *The incidence on producers of increasing t^S is*

$$\frac{dp}{dt^S} = \frac{\partial D/\partial t^S}{\partial S/\partial p - \partial D/\partial p} = -\frac{\varepsilon_{D,q|t^S}}{\frac{q}{p}\varepsilon_{S,p} + \varepsilon_{D,q|p}} = -\frac{\theta\varepsilon_{D,q|p}}{\frac{q}{p}\varepsilon_{S,p} + \varepsilon_{D,q|p}} \tag{2}$$

and the incidence on consumers is

$$\frac{dq}{dt^S} = 1 + \frac{dp}{dt^S} = \frac{\frac{q}{p}\varepsilon_{S,p} + \varepsilon_{D,q|p} - \varepsilon_{D,q|t^S}}{\frac{q}{p}\varepsilon_{S,p} + \varepsilon_{D,q|p}} = \frac{\frac{q}{p}\varepsilon_{S,p} + (1-\theta)\varepsilon_{D,q|p}}{\frac{q}{p}\varepsilon_{S,p} + \varepsilon_{D,q|p}}$$

where $\partial D/\partial t^S$ and $\partial D/\partial p$ are both evaluated at $(p + t^E, t^S, Z)$ and $\partial S/\partial p$ is evaluated at p.

Figure 1 illustrates the incidence of introducing a sales tax t^S in a market that is initially untaxed. The figure plots supply and demand as a function of the pretax price p. The market initially clears at a price $p_0 = p(0, 0)$. When the tax is levied, the demand curve

[7] The literature in psychology and economics has argued that firms are less prone to systematic errors than consumers (see e.g. section IV of DellaVigna 2007). It would be straightforward to extend the analysis to allow for salience effects on the firm side as well, in which case the formulas will depend on $\frac{\partial S}{\partial p}$ and $\frac{\partial S}{\partial t^S}$.

shifts inward by $t^S \partial D/\partial t^S$ units, creating an excess supply of $E = t^S \partial D/\partial t^S$ units of the good at the initial price p_0. To re-equilibrate the market, producers cut the pretax price by $E/(\partial S/\partial p - \partial D/\partial p)$ units. The only difference in the incidence diagram in Figure 1 relative to the traditional model without salience effects is that the demand curve shifts inward by $t^S \partial D/\partial t^S$ instead of $t^S \partial D/\partial p$. With salience effects, the shift in the demand curve is determined by the tax elasticity, while the price adjustment needed to clear the market is determined by the price elasticity. This is why one must estimate *both* the tax and price elasticities to calculate incidence.

Three general lessons about tax incidence emerge from the formulas in Proposition 1.

1. [Attenuated Incidence on Producers] Incidence on producers is attenuated by $\theta = \frac{\partial D/\partial t^S}{\partial D/\partial p}$ relative to the traditional model. Intuitively, producers face less pressure to reduce the pretax price when consumers under react to the sales tax. In the extreme case where $\partial D/\partial t^S = 0$, consumers bear all of the tax, because there is no need to change the pretax price to clear the market. More generally, the incidence of a tax on consumers is inversely related to the degree of attention to the tax (θ).

One interpretation of this result is that the demand curve becomes more inelastic when individuals are inattentive. Though changes in inattention and the price elasticity both affect the gross-of-tax-elasticity $\varepsilon_{D,q|t^S} = \theta \varepsilon_{D,q|p}$ in the same way, their effects on incidence are not equivalent. To see this, consider two markets, A and B, where $\varepsilon^A_{S,p} = \varepsilon^B_{S,p} = 0.1$. In market A, demand is inelastic and consumers are fully attentive to taxes: $\varepsilon^A_{D,q|p} = 0.3$ and $\theta^A = 1$. In market B, demand is elastic but consumers are inattentive: $\varepsilon^B_{D,q|p} = 1$ and $\theta^B = 0.3$. An econometrician would estimate the same tax elasticity in both markets: $\varepsilon^A_{D,q|t^S} = \varepsilon^B_{D,q|t^S} = 0.3$. However, $[\frac{dp}{dt^S}]^A = -0.75$ whereas $[\frac{dp}{dt^S}]^B = -0.27$. In market A, suppliers bear most of the incidence since demand is 3 times more elastic to price than supply. In market B, even though demand is 10 times as price elastic as supply, producers are able to shift most of the incidence of the tax to consumers because of inattention.

Intuitively, a low price elasticity of demand has two effects on incidence: it reduces the shift in the demand curve but increases the size of the price cut needed to re-equilibrate the market for a given level of excess supply. Inattention to the tax also reduces the shift in the demand curve, but does not have the second offsetting effect. This difference is apparent in the formula for $\frac{dp}{dt}$ in (2), where $\varepsilon_{D,q|p}$ appears in both the numerator and denominator whereas θ appears only in the numerator. As a result, a 1 percent reduction in attention leads to greater incidence on consumers than a 1 percent reduction in the price elasticity. As $\varepsilon_{S,p}$ approaches 0, $\frac{dq}{dt^S}$ approaches $1-\theta$ irrespective of $\varepsilon_{D,q|p}$. If consumers are sufficiently inattentive, they bear most of the incidence of a tax even if supply is inelastic.

2. [No Tax Neutrality] Taxes that are included in posted prices have greater incidence on producers because they are fully salient: $\frac{dp}{dt^E} = \frac{\partial D/\partial t^S}{\partial S/\partial p - \partial D/\partial p} < \frac{dp}{dt^S}$. Taxes levied on producers are more likely to be included in posted prices than taxes levied on consumers because producers must actively "shroud" a tax levied on them in order to reduce its salience. Together, these observations suggest that producers will generally bear more of the incidence when a tax is levied on them than when it is levied on the consumers. Statutory incidence affects economic incidence, contrary to intuition based on the full-optimization model.[8]

3. [Effect of Price Elasticity] Holding fixed the size of the tax elasticity $\varepsilon_{D,q|t^S}$, an increase in the price elasticity of demand *raises* incidence on consumers ($\partial[\frac{dp}{dt^S}]/\partial\varepsilon_{D,q|p} > 0$). This is because holding fixed the shift in the demand curve created by the introduction of the tax, a smaller price reduction is needed to clear the market if demand is very price elastic. In contrast, if the degree of inattention θ is held fixed as $\varepsilon_{D,q|p}$ varies, one obtains the conventional result $\partial[\frac{dp}{dt^S}]/\partial\varepsilon_{D,q|p} < 0$ because $\varepsilon_{D,q|t^S}$ and $\varepsilon_{D,q|p}$ vary at the same rate. Thus, taxing markets with more elastic demand could lead to greater or lesser incidence on consumers, depending on the extent to which the tax elasticity $\varepsilon_{D,q|t^S}$ covaries with the price elasticity $\varepsilon_{D,q|p}$.

III Efficiency Cost

I begin by characterizing the excess burden of introducing a sales tax t^S in an initially untaxed market with constant-returns-to-scale production (fixed producer prices). I then

[8] Consistent with this prediction, Busse, Silva-Risso, and Zettelmeyer (2006) find that 35 percent of manufacturer rebates given to car dealers are passed through to the buyer, while 85 percent of rebates given to buyers stay with the buyer. The reason is that most consumers did not find out about the dealer rebates. Rudolf Kerschbamer and Georg Kirchsteiger (2000) find that statutory evidence affects economic incidence in a lab experiment.

extend the analysis to allow for endogenous producer prices and pre existing excise and sales taxes.

III.A Definitions

I first define generalized indirect utility and expenditure functions that permit prices and taxes to have different effects. Let $V(p+t^E,t^S,Z) = u(x(p+t^E,t^S,Z)) + v(y(p+t^E,t^S,Z))$ denote the agent's indirect utility as a function of the posted price of good x, the sales tax, and wealth. Let $e(p+t^E,t^S,V)$ denote the agent's expenditure function, which represents the minimum wealth necessary to attain utility V at a given posted price and sales tax. Let $R(t^E,t^S,Z) = tx(p+t^E,t^S,Z)$ denote tax revenue.

Following Herbert Mohring (1971) and Alan J. Auerbach (1985), I measure excess burden using the concept of equivalent variation. When p is fixed, the excess burden of introducing a sales tax t^S in a previously untaxed market is

$$EB(t^S) = Z - e(p,0,V(p,t^S,Z)) - R(0,t^S,Z) \tag{3}$$

The value $EB(t^S)$ is the amount of additional tax revenue that could be collected from the consumer while keeping his utility constant if the distortionary tax were replaced with a lump-sum tax. Roughly speaking, $EB(t^S)$ can be interpreted as the total value of the purchases that fail to occur because of the tax. The objective is to derive approximate expressions for (3) in terms of empirically estimable elasticities.

III.B Preference Recovery

The efficiency cost of a tax policy depends on two elements: (1) the change in behavior induced by the tax and (2) the effect of that change in behavior on the consumer's utility. The first element is observed empirically – one can estimate the demand function $x(p+t^E,t^S,Z)$. The second element is the key challenge for behavioral welfare economics. How does one compute indirect utility $V(p+t^E,t^S,Z)$ when the agent's behavior is not consistent with optimization? The following two assumptions allow us to recover V without specifying a positive model for the demand function $x(p+t^E,t^S,Z)$.

A1 Taxes affect utility only through their effects on the chosen consumption bundle. The agent's indirect utility given taxes of (t^E, t^S) is

$$V(p + t^E, t^S, Z) = u(x(p + t^E, t^S, Z)) + v(y(p + t^E, t^S, Z))$$

A2 When tax-inclusive prices are fully salient, the agent chooses the same allocation as a fully-optimizing agent:

$$x(p, 0, Z) = x^*(p, 0, Z) = \arg\max u(x(p, 0, Z)) + v(Z - px(p, 0, Z))$$

Assumption A1 requires that consumption is a sufficient statistic for utility – that is, holding fixed the consumption bundle (x, y), the tax rate or its salience has no effect on V. To understand the content of this assumption, consider the following situation in which it is violated. In a bounded rationality model, the cognitive cost that the agent pays to calculate the total price when $t^S > 0$ makes his utility lower than pure consumption utility. Taxes that are not included in posted prices therefore generate deadweight burden beyond that due to the distortion in the consumption bundle (Chetty, Looney, and Kroft 2007). In such models, the excess burden computations in this paper correspond to the deadweight cost net of any increase in cognitive costs.[9]

Assumption A2 requires that the agent behaves like a fully-optimizing agent when all taxes are fully salient. That is, the agent's choices when total prices are fully salient reveal his true rankings. This assumption is violated when the agent's choices are suboptimal even without taxes. For example, if there are other "shrouded attributes" or if agents suffer from biases when optimizing relative to prices (Nina Mazar, Botond Koszegi, and Dan Ariely 2008), one would not directly recover true preferences from $x(p, 0, Z)$. The excess burden formulas derived below ignore errors in optimization relative to prices.

Using assumptions A1 and A2, I calculate consumer welfare and excess burden in two steps. I first use the demand function without taxes $x(p, 0, Z)$ to recover the agent's underlying preferences $(u(x), v(y))$ as in the full-optimization model. I then use the demand

[9] Chetty, Looney, and Kroft (2007) show that the additional deadweight burden due to cognitive costs is likely to be negligible since relatively small cognitive costs generate substantial amounts of inattention.

function with taxes $x(p+t^E, t^S, Z)$ to calculate the agent's indirect utility $V(p+t^E, t^S, Z)$ as a function of the tax rate. Conceptually, this method pairs the libertarian criterion of calculating welfare from individual choice with the assumption that the agent optimizes relative to true incentives only when tax-inclusive prices are perfectly salient.

This calculation of excess burden can be viewed as an application of Bernheim and Rangel's (2009) choice-based approach to welfare analysis. Bernheim and Rangel show that one can obtain bounds on welfare without specifying a positive theory of behavior by separating the inputs that matter for utility from "ancillary conditions" that do not. By applying a "refinement" to identify ancillary conditions under which an agent's choices reveal his true rankings, one can sharpen the bounds. In Bernheim and Rangel's terminology, assumption A1 is that tax salience is an "ancillary condition" that affects choices but not true utility. Assumption A2 is a "refinement" which posits that the choices made when the tax is not perfectly salient are "suspect," and should be discarded when inferring the utility relevant for welfare analysis. This refinement allows us to obtain exact measures of equivalent variation and efficiency costs without placing structure on the model that generates $x(p+t^E, t^S, Z)$.

III.C Fixed Producer Prices

I derive analytical formulas for excess burden using approximations analogous to those used by Harberger (1964) and Edgar K. Browning (1987). Like the widely applied Harberger-Browning formula, the formulas below ignore the third- and higher-order terms in the Taylor expansion for excess-burden. Hence, the formulas provide accurate measures of excess burden for small tax changes.

In this section, I characterize excess burden of introducing a sales tax in a market where production is constant-returns-to-scale ($c'' = 0$). In this case, the pretax price of x is fixed at $p = c'(0)$ because the supply curve is flat. Moreover, since firms earns zero profits ($\pi = 0$), only consumer welfare matters for excess burden. To state the formula compactly, let us introduce the following notation for income-compensated elasticities. Let $\partial x^c/\partial p = \partial x/\partial p + x\partial x/\partial Z$ denote the income-compensated (Hicksian) price effect. Define $\partial x^c/\partial t^S = \partial x/\partial t^S + x\partial x/\partial Z$ as the analogous income-compensated tax effect. Note that this "compensated tax effect" does not necessarily satisfy the Slutsky condition $\partial x^c/\partial t^S < 0$. It is possible to have an upward-sloping compensated tax-demand curve because $x(p, t^S, Z)$ is not generated by utility maximization. In contrast, assumption A2 guarantees $\frac{\partial x^c}{\partial p} < 0$ through the Slutsky condition. Let $\varepsilon^c_{x,q|p} = -\frac{\partial x^c}{\partial p}\frac{q}{x}$ and $\varepsilon^c_{x,q|t^S} = -\frac{\partial x^c}{\partial t^S}\frac{q}{x}$ denote the compensated price and tax elasticities.

Proposition 2 *Suppose producer prices are fixed ($\varepsilon_{s,p} = \infty$). Under assumptions A1-A2, the excess burden of introducing a small tax t^S in an untaxed market is approximately*

$$\begin{aligned} EB(t^S) &\simeq -\frac{1}{2}(t^S)^2\theta^c\partial x^c/\partial t^S \\ &= \frac{1}{2}(t^S)^2\theta^c x(p, t^S, Z)\frac{\varepsilon^c_{x,q|t^S}}{p + t^S} \end{aligned} \tag{4}$$

where $\partial x^c/\partial t^S$ and $\partial x^c/\partial p$ are evaluated at $(p, 0, Z)$ and $\theta^c = \frac{\partial x^c/\partial t^S}{\partial x^c/\partial p} = \frac{\varepsilon^c_{x,q|t^S}}{\varepsilon^c_{x,q|p}}$ is the ratio of the compensated tax and price effects.

Proof. Here, I provide an instructive proof for the case without income effects ($\frac{\partial x}{\partial Z} = 0$), which implies that utility is quasilinear ($v(y) = y$). The derivation for the general case is given in Appendix A. To reduce notation, I suppress wealth and write $x(p, t^S)$.

First use assumption A1 to obtain an expression for indirect utility:

$$\begin{aligned} V(p, t^S, Z) &= u(x(p, t^S) + y(p, t^S) = u(x(p, t^S)) + Z - (p + t^S)x(p, t^S) \\ &= Z + u(0) + \int_0^{x(p,t^S)} [u'(x) - (p + t^S)]dx \end{aligned} \tag{5}$$

Recognizing that e is the inverse of V and using (3), it follows that excess burden is

$$EB(t^S) = \int_{x(p,0)}^{x(p,t^S)} [u'(x) - p]dx.$$

To recover $u'(x)$ empirically, use A2, which implies that

$$\begin{aligned} u'(x(p, 0)) &= u'(x^*(p, 0)) = p \\ \Rightarrow u'(x) &= P(x) \end{aligned}$$

where $P(x) = x^{-1}(p, 0)$ is the inverse price-demand function. It follows that

$$EB(t^S) = \int_{x(p,0)}^{x(p,t^S)} [P(x) - p]dx \tag{6}$$

which measures the area under the inverse-demand curve between $x(p, 0)$ and $x(p, t^S)$. This is an exact formula for excess burden that could be implemented with a non-parametric estimate of the demand curve. A simple analytical formula can be obtained by approximating $EB(t^S)$ using a Taylor expansion. I ignore the $(t^S)^3$ and higher-order terms, which is equivalent to assuming that $x(\cdot)$ is linear when utility is quasilinear. Evaluating the integral in (6) with this approximation yields

$$EB(t^S) \simeq -\frac{1}{2}(t^S)^2 \frac{\partial x/\partial t^S}{\partial x/\partial p} \partial x/\partial t^S \tag{7}$$

which corresponds to (4) because compensated and uncompensated elasticities are equal when $\partial x/\partial Z = 0$.

Graphical Derivation. Figure 2 illustrates the calculation of deadweight loss for the case without income effects. The initial price of the good is p_0 and the price after the imposition of the sales tax is $p_0 + t^S$. The figure plots two demand curves. The first is the standard Marshallian demand curve as a function of the total price of the good, $x(p, 0)$. This *price-demand* curve coincides with the marginal utility $u'(x)$ as shown in the proof above. The second, $x(p_0, t^S)$ represents how demand varies with the tax on x. This *tax-demand* curve is drawn assuming $\partial x/\partial p \leq \partial x/\partial t^S$, consistent with the empirical evidence.

The agent's initial consumption choice prior to the introduction of the tax is depicted by $x_0 = x(p_0, 0)$. Initial consumer surplus is given by triangle ABC, which equals total utility (up to a constant) as shown by (5). When the tax t^S is introduced, the agent cuts consumption of x by $\Delta x = -t^S \partial x/\partial t^S$. Notice that at the new consumption choice x_1, the agent's marginal willingness-to-pay for x is below the total price $p_0 + t^S$ because he under-reacts to the tax. This optimization error leads to a loss of surplus corresponding to triangle DEF. The consumer's surplus after the implementation of the tax is therefore given by triangle DGC minus triangle DEF. The revenue raised from the tax corresponds to the rectangle $GBEH$. It follows that the change in total surplus – government revenue plus consumer surplus – equals the shaded triangle AFH. This is precisely the measure in (6) – the area under the price-demand curve between x_0 and x_1. The base of the triangle (AH) has length $-t^S \frac{\partial x}{\partial t^S}$ while the height of the triangle (AF) is $t^S \frac{\partial x/\partial t^S}{\partial x/\partial p}$, yielding (7).[10]

When there are income effects ($\frac{\partial x}{\partial Z} > 0$), the form of the formula remains exactly the same, but all the inputs are replaced by income-compensated effects, exactly as in the Harberger formula. The intuition for this difference is analogous to that in the full-optimization model: behavioral responses due to pure income effects are non-distortionary, since they would occur under lump sum taxation as well. Deadweight loss is determined by difference between the actual behavioral response ($\frac{\partial x}{\partial t^S}$) and the socially optimal response given the reduction in net-of-tax income ($-x\frac{\partial x}{\partial Z}$), which is $\frac{\partial x}{\partial t^S} - (-x\frac{\partial x}{\partial Z}) = \frac{\partial x^c}{\partial t^S}$.[11]

Note that in Proposition 2 and all subsequent excess burden calculations, $\partial x^c/\partial p$ is evaluated at a point with zero sales tax $(p, 0)$. The reason is that one recovers true preferences only when the posted price equals the total price: $x(p, t^S, Z) = x^*(p, t^S, Z)$ if and only if $t^S = 0$. If an environment without sales tax is not observed, one could implement the formula by assuming that the price elasticity does not depend on the tax rate ($\frac{d^2x^c}{dpdt^S} = 0$), a plausible assumption for small tax rates. Under this assumption, $\frac{dx^c}{dp}(p, 0, Z) = \frac{dx^c}{dp}(p, t^S, Z)$, which can be estimated empirically as in Chetty, Looney, and Kroft (2007).

Discussion. The only difference between (4) and the canonical Harberger formula ($EB^*(t^S) = -\frac{1}{2}(t^S)^2\frac{\partial x^c}{\partial t^S}$) is the ratio of the tax and price effects $\frac{\partial x^c/\partial t^S}{\partial x^c/\partial p}$. Three general lessons about excess burden emerge from this ratio.

1. [Inattention Reduces Excess Burden if $\frac{\partial x}{\partial Z} = 0$] When there are no income effects, the tax t^S generates deadweight cost equivalent to that created by a perfectly salient tax of θt^S. If agents ignore taxes completely and $\theta = 0$, then $EB = 0$. Taxation creates no inefficiency when $\theta = 0$ because the agent's consumption allocation coincides with the first-best bundle

[10] Another instructive derivation starts from the excess burden of taxation for a fully-optimizing agent, EB^* (triangle AID). Starting from EB^*, I obtain excess burden for the agent who does not optimize fully (triangle AFH) by making two adjustments: (1) subtracting the additional revenue earned by the government because the agent under-reacts to the tax (rectangle $HIDE$) and (2) adding the private welfare loss due to the optimization error (triangle FED).

[11] Income effects have more complex effects on the excess burden calculation when there are more than two goods because the tax may create a suboptimal budget allocation among the untaxed goods.

that he would have chosen under lump sum taxation.[12] As the degree of attention to taxes rises, excess burden rises at a quadratic rate: $EB \propto \theta^2$. Excess burden rises with the square of θ for the same reason that it rises with the square of the t^S – the increasing marginal social cost of deviating from the first-best. Because EB is a quadratic function of θ but a linear function of $\varepsilon_{x,q|p}$, inattention (reductions in θ) and inelasticity (reductions in $\varepsilon_{x,q|p}$) have different effects on excess burden, as in the incidence analysis. Like incidence, excess burden depends on which side of the market is taxed. Since a tax on producers is likely to be included in posted prices, it leads to a larger reduction in demand and more deadweight loss than an equivalent tax levied on consumers when $\frac{\partial x}{\partial Z} = 0$.

2. [Inattention Can Raise Excess Burden if $\frac{\partial x}{\partial Z} > 0$] When there are income effects, making a tax less salient to reduce $\frac{\partial x}{\partial t^S}$ can *increase* deadweight loss. In fact, a tax can create deadweight cost even if the agent completely ignores it and demand for the taxed good does not change, i.e. $\frac{\partial x}{\partial t^S} = 0$. This result contrasts with the canonical intuition that taxes generate deadweight costs only if they induce changes in demand. In the full-optimization model, taxation of a normal good creates a deadweight cost only if $\frac{\partial x}{\partial p} < 0$ since $\frac{\partial x}{\partial p} = 0 \Rightarrow \frac{\partial x^c}{\partial p} = 0$ given $\frac{\partial x}{\partial Z} > 0$. This reasoning fails when the tax-demand is not the outcome of perfect optimization, because there is no Slutsky condition for $\frac{\partial x^c}{\partial t^S}$. A zero uncompensated tax elasticity does not imply that the compensated tax elasticity is zero. Instead, when $\frac{\partial x}{\partial t^S} = 0$, $\frac{\partial x^c}{\partial t^S} = \frac{\partial x}{\partial Z}$ and (4) becomes

$$EB(t^S) = -\frac{1}{2}(t^S x)^2 \frac{\partial x/\partial Z}{\partial x^c/\partial p} \partial x/\partial Z$$

This equation shows that $EB > 0$ even when $\partial x/\partial t^S = 0$ in the presence of income effects. To understand this result, recall that the excess burden of a distortionary tax is determined by the extent to which the agent deviates from the allocation he would optimally choose if subject to a lump sum tax of an equivalent amount. In the quasilinear case, the agent's consumption bundle when ignoring the tax coincides with the bundle he would optimally

[12] The consumer's *private* welfare always rises with θ – increased salience of tax-inclusive prices is always desirable from the consumer's perspective. However, the gain in the consumer's private welfare from full attention (triangle FED in Figure 4) is more than offset by the resulting loss in government revenue (rectangle $HIDE$), which is why total surplus falls with θ when $\frac{\partial x}{\partial Z} = 0$.

choose under lump sum taxation, because the socially optimal choice of x does not depend on total income. When utility is not quasilinear, an optimizing agent would reduce consumption of both x and y when faced with a lump sum tax. An agent who does not change his demand for x at all when the tax is introduced ends up over-consuming x relative to the social optimum. The income-compensated tax elasticity $\frac{\partial x^c}{\partial t^S} = \frac{\partial x}{\partial Z}$ is positive because the tax effectively distorts demand for x *upward* once the income effect is taken into account, leading to inefficiency.

As a concrete example, consider an individual who consumes cars (x) and food (y). Suppose he chooses the same car he would have bought at a total price of p_0 because he does not perceive the tax ($\frac{\partial x}{\partial t^S} = 0$) and therefore has to cut back on food to meet his budget. This inefficient allocation of net-of-tax income leads to a loss in surplus. The lost surplus is proportional to the income effect on cars $\frac{\partial x}{\partial Z}$ because this elasticity determines how much the agent should have cut spending on the car to reach the social optimum given the tax. This example illustrates that policies which "hide" taxes can potentially create substantial deadweight loss despite attenuating behavioral responses, particularly when the income elasticity and expenditure on the taxed good are large.

Note that inattention to a tax on x need not necessarily lead to $\frac{\partial x}{\partial t^S} = 0$. The effect of inattention on $\frac{\partial x}{\partial t^S}$ depends on how the agent meets his budget given the tax. The agent must reduce consumption of at least one of the goods to meet his budget when the tax on x is introduced: $\frac{\partial x}{\partial t^S} + \frac{\partial y}{\partial t^S} = -x$. The way in which agents meet their budget may vary across individuals (Chetty, Looney, and Kroft 2007). For example, credit-constrained agents may be forced to cut back on consumption of y if they ignore the tax when buying x, as in the car purchase example above, leading to $\frac{\partial x}{\partial t^S} = \theta = 0$ and $EB > 0$. Agents who smooth intertemporally, in contrast, may cut both y as well as future purchases of x (buying a cheaper car next time). Such intertemporal smoothing could lead to a long-run allocation closer to the socially optimal $\frac{\partial x}{\partial t^S} = -x\frac{\partial x}{\partial Z}$, in which case hidden taxes would lead to $\theta^c = 0$ and $EB = 0$. Importantly, Proposition 2 holds irrespective of how the agent meets his budget. Variations in the budget adjustment process are captured in the value of $\frac{\partial x^c}{\partial t^S}$.

3. [Role of Price Elasticity] Holding fixed $\varepsilon_{x,q|t^S}$, excess burden is inversely related to $\varepsilon_{x,q|p}$. As demand becomes *less* price-elastic, *EB increases.* This can be seen in Figure 2,

where the shaded triangle becomes larger as $x(p, 0)$ becomes steeper, holding $x(p_0, t^S)$ fixed. Intuitively, an agent with price-inelastic consumption has rapidly increasing marginal utility as his consumption level deviates from the first-best level. A given reduction in demand thus leads to a larger loss of surplus for an agent with more price-inelastic demand. As in the incidence analysis, taxing markets with more elastic demand could lead to greater or lesser excess burden, depending on the covariance between $\varepsilon_{x,q|t^S}$ and $\varepsilon_{x,q|p}$.

III.D Endogenous Producer Prices

I now drop the constant-returns-to-scale assumption and consider a market where the supply curve is upward sloping ($\varepsilon_{S,p} < \infty$). In this case, pretax prices are endogenous to the tax rate and firms earn positive profits, which must be accounted for in the welfare calculation. Following Auerbach (1985), assume that profits $\pi(S(p))$ are paid to the consumer using the numeraire y. In this subsection, I assume that utility is quasilinear ($v(y) = y$). I do not treat the case with both income effects and $\varepsilon_{S,p} < \infty$ in this paper.[13] When $p(t_0^E, t_0^S)$ is endogenous and $\frac{\partial x}{\partial Z} = 0$, excess burden is

$$EB(t^S) = Z - e(p_0, 0, V(p_1, t^S, Z)) + \pi_0 - \pi_1 - R$$

where $p_0 = p(0, 0)$ and $p_1 = p(0, t^S)$ denote the equilibrium price before and after the introduction of the tax, $\pi_i = \pi(S(p_i))$, and $R = t^S x(p_1, t^S)$ denotes tax revenue (Auerbach 1985). Intuitively, excess burden equals the sum of the change in consumer surplus and producer surplus minus government revenue (R). Let $\frac{dx}{dt^S}$ denote the total reduction in equilibrium quantity caused by the tax, taking into account the effect of the price response: $\frac{dx}{dt^S} = \frac{\partial x}{\partial p}\frac{\partial p}{\partial t^S} + \frac{\partial x}{\partial t^S}$. Correspondingly, let $\varepsilon_{x,q|t^S}^{TOT} = -\frac{dx}{dt^S}\frac{q}{x(p,t^S)}$ denote the total change in demand caused by a 1 percent increase in the price $q = p_1 + t^S$ through an increase in t^S, taking into account the effect of the endogenous price response.

Proposition 3 *Suppose utility is quasilinear ($v(y) = y$). Under assumptions A1-A2, the*

[13] Even in the full-optimization model, analytical formulas for excess burden cannot be obtained when there are income effects and non-zero producer surplus (Auerbach 1985).

excess burden of introducing a small tax t^S in a previously untaxed market is approximately

$$\begin{aligned} EB(t^S) &\simeq -\frac{1}{2}(t^S)^2\theta\frac{dx}{dt^S} \\ &= \frac{1}{2}(t^S)^2\theta x(p_1,t^S)\frac{\varepsilon^{TOT}_{x,q|t^S}}{p_1+t^S}. \end{aligned} \tag{8}$$

where $\frac{dx}{dt^S}$ is evaluated at $(p_0, 0, Z)$ and $\theta = \frac{\partial x/\partial t^S}{\partial x/\partial p} = \frac{\varepsilon_{x,q|t^S}}{\varepsilon_{x,q|p}}$.

Proof. The equation can be derived heuristically by calculating the area of the triangle that lies between the supply and the (no tax) price-demand curve $x(p,0)$ between the initial and final equilibrium quantities in Figure 1. The width of the triangle is $t^S\frac{dx}{dt^S}$ and the height is θt^S. See Appendix A for a formal derivation.

The lessons discussed above with fixed producer prices carry over to the case with endogenous prices. Indeed, the formula for excess burden with upward-sloping supply has exactly the same form as in (4). The only difference is that the size of the deviation in demand from the social optimum is given by the total derivative $\frac{dx}{dt^S}$ instead of the partial derivative $\frac{\partial x}{\partial t^S}$. When p is fixed, these two derivatives coincide, so (8) collapses to the formula in Proposition 2 without income effects. When p is endogenous, part of the distortion in behavior is offset by the reduction in prices by producers to clear the market, leading to $|\frac{dx}{dt^S}| < |\frac{\partial x}{\partial t^S}|$ and smaller deadweight loss.

III.E Preexisting Taxes

Finally, I calculate the marginal deadweight cost of increasing the sales tax by Δt when there are preexisting taxes on good x. The initial excise tax rate is t^E_0 and sales tax rate is t^S_0. Let $p_0 = p(t^E_0, t^S_0)$ denote the initial pre-tax equilibrium price, $q_0 = p_0 + t^E_0 + t^S_0$ denote the initial tax-inclusive price, and $x_0 = x(p_0 + t^E_0, t^S_0)$ denote initial quantity sold. Let $p^E_0 = p(t^E_0, 0)$ denote the price when there is only an excise tax and $p_1 = p(t^E_0, t^S_0 + \Delta t)$ denote the price after the tax increase. The following proposition provides approximate formulas for excess burden that are accurate for small initial tax rates (t^E_0, t^S_0) and a small tax increase Δt. I explain why the approximation requires small initial tax rates after stating the result, and provide a formal statement of this requirement in Appendix A.

Proposition 4 *Under assumptions A1-A2, the excess burden of a small sales tax increase Δt starting from small initial tax rates (t_0^E, t_0^S) is approximately given by the following formulas, with all derivatives evaluated at the no-sales-tax equilibrium $(p_0^E + t_0^E, 0)$.*
(i) If producer prices are fixed ($\varepsilon_{s,p} = \infty$):

$$\begin{aligned} EB(\Delta t|t_0^E, t_0^S) &\simeq -\frac{1}{2}(\Delta t)^2\theta^c\frac{\partial x^c}{\partial t^S} - \Delta t\frac{\partial x^c}{\partial t^S}[t_0^E + \theta^c t_0^S] \\ &= \frac{1}{2}(\Delta t)^2\theta^c x_0\frac{\varepsilon^c_{x,q|t^S}}{q_0} + \Delta t x_0\frac{\varepsilon^c_{x,q|t^S}}{q_0}[t_0^E + \theta^c t_0^S] \end{aligned}$$

(ii) If utility is quasilinear ($v(y) = y$):

$$\begin{aligned} EB(\Delta t|t_0^E, t_0^S) &\simeq -\frac{1}{2}(\Delta t)^2\theta\frac{dx}{dt^S} - \Delta t\frac{dx}{dt^S}[t_0^E + \theta t_0^S] \\ &= \frac{1}{2}(\Delta t)^2\theta x_0\frac{\varepsilon^{TOT}_{x,q|t^S}}{q_0} + \Delta t x_0\frac{\varepsilon^{TOT}_{x,q|t^S}}{q_0}[t_0^E + \theta t_0^S]. \end{aligned}$$

Proof. See Appendix A and Appendix Figure 1.

The first term in these formulas, proportional to $(\Delta t)^2$, is analogous to the triangle in the classic "Harberger trapezoid." This term comes from the loss in consumer and producer surplus due to the tax increase, and is exactly the same as in the case without preexisting taxes. The second term, proportional to Δt, is analogous to the rectangle in the Harberger trapezoid. This term reflects the fiscal externality that the agents impose on the government by changing their behavior. Government revenue falls by $\Delta t\frac{dx^c}{dt^S}[t_0^E + t_0^S]$ because of the behavioral response to the tax increase. However, part of this fiscal externality is offset by a gain of $\Delta t\frac{dx^c}{dt^S}[1 - \theta^c]t_0^S$ in private utility to the consumer, since he was initially over consuming x relative to his private optimum.

With preexisting taxes, tax increases can have a first-order (large) deadweight cost. The first-order deadweight cost due to t_0^S is multiplied by θ^c because the deviation from the socially optimal level of x caused by t_0^S is proportional to θ^c. If utility is quasilinear, levying a tax on top of a preexisting tax t_0^S that is completely hidden ($\theta = \theta^c = 0$) generates only second-order (small) excess burden. If utility is not quasilinear, the same tax increase generates a first-order deadweight cost. Intuitively, the agent's consumption bundle is distorted relative to the social optimum to begin with if $\frac{dx}{dt^S} = 0$ when there are income effects. An increase in the tax rate exacerbates this preexisting distortion, creating a first-order deadweight cost even though there is no change in uncompensated demand.

I close with a technical remark about the approximations used in Proposition 4. The classic "Harberger trapezoid" formula requires that Δt is small and that either (1) initial tax rates are small or (2) demand is linear ($\frac{d^2x}{dp^2} = 0$) over the Δt interval (Auerbach 1985). In the case studied here, for small Δt, condition (1) suffices to obtain simple formulas for EB but (2) does not. The reason is that one can only recover the utility of $x(p, t^S)$ when $t^S = 0$ under A2. To calculate $V(p, t_0^S)$, I assume that t_0^S is small and take a Taylor expansion around $V^*(p, t_0^S)$, ignoring the third- and higher-order terms. Linearity over the Δt interval itself does not allow us to calculate $V(p, t_0^S)$ when $t_0^S > 0$.[14] Note that all of the approximations in Propositions 2-4 are needed only to obtain simple analytical formulas for EB. Exact measures of excess burden can be calculated using a non-parametric estimate of $x(p, t)$, as in the full-optimization model (Jerry A. Hausman and Whitney K. Newey 1995).

IV Conclusion

A growing body of evidence shows that individuals optimize imperfectly with respect to many tax and transfer policies. The formulas developed in this paper can be applied to characterize the incidence and efficiency costs of such policies. Much as Harberger identified the compensated price elasticity as the key parameter to be estimated in subsequent work, the analysis here identifies the compensated tax and price elasticities ($\varepsilon^c_{x,q|t^S}$ and $\varepsilon^c_{x,q|p}$) as "sufficient statistics" for welfare analysis in behavioral models of tax policy.

A natural next step would be to extend the welfare analysis in this paper to characterize optimal taxation when agents optimize imperfectly, generalizing the results of Ramsey (1927) and Mirrlees (1971). Combining the formulas here with a positive theory would be useful for this analysis. For example, Chetty, Looney, and Kroft's (2007) bounded-rationality model predicts that attention and behavioral responses to taxation are larger when (1) tax rates

[14] Linearity of demand over the Δt interval permits large t_0^E but not t_0^S. To allow for large t_0^S, one must make additional parametric assumptions about the demand and utility functions between $x(p_0^E + t_0^E, 0)$ and $x(p_0 + t_0^E, t_0^S)$ to calculate $V(p_0 + t_0^E, t_0^S)$.

are high, (2) the price-elasticity of demand is large, and (3) the amount spent on the good is large. Combined with the welfare analysis here, these predictions imply that in markets with these three characteristics, tax incidence should fall more heavily on producers and excess burden should be closer to the Harberger measure.

Finally, the approach to welfare analysis proposed here – using a domain where incentives are fully salient to characterize the welfare consequences of policies that are not salient – can be applied in other contexts. Many social insurance programs (e.g. Medicare and Social Security) have complex features and may induce suboptimal behaviors. By estimating behavioral responses to analogous programs whose incentives are more salient, one can characterize the welfare consequences of the existing programs more accurately. Another potential application is to optimal regulation, including consumer protection law and financial market regulations. By identifying "suboptimal" transactions using data on consumer's choices in domains where incentives are more salient, one could develop rules to maximize consumer welfare that do not rely on paternalistic judgments.[15]

[15] For instance, the terms stated on the first page of a contract are likely to be more salient than those in fine print or in later pages of a contract. By comparing how behavior responds to incentives stated on the front page vs. other parts of the contract, one may be able to gauge the welfare losses of complexity and the benefits of regulation.

References

[1] **Atkinson, Anthony B. and Joseph E. Stiglitz**. 1976. "The design of tax structure: direct versus indirect taxation." *Journal of Public Economics*, 6(1): 55-75.

[2] **Auerbach, Alan J.** 1985. "The Theory of Excess Burden and Optimal Taxation," in *Handbook of Public Economics* vol. 1, ed. Alan Auerbach and Martin Feldstein, 61-128. Amsterdam: Elsevier Science Publishers B. V.

[3] **Bernheim, B. Douglas and Antonio Rangel**. 2009. "Beyond Revealed Preference: Choice-Theoretic Foundations for Behavioral Welfare Economics." *Quarterly Journal of Economics*, 124(1): 51–104.

[4] **Blendon, Robert J., Stephen R. Pelletier, Marcus D. Rosenbaum, and Mollyann Brodie.** 2003. "Tax Uncertainty." *Brookings Review* (Summer): 28-31.

[5] **Blumkin, Tomer, Bradley J. Ruffle, and Yosi Ganun.** 2008. "Are Income and Consumption Taxes Ever Really Equivalent? Evidence from a Real-Effort Experiment." University Library of Munich, Germany, MPRA Paper 6479.

[6] **Browning, Edgar K. 1987**. "On The Marginal Welfare Cost of Taxation." *American Economic Review*, 77: 11-23.

[7] **Busse, Meghan, Jorge Silva-Risso and Florian Zettelmeye**r. 2006. "$1000 Cash Back: The Pass-Through of Auto Manufacturer Promotions." *American Economic Review*, 96(4): 1253-1270.

[8] **Chetty, Raj**. 2006. "A New Method of Estimating Risk Aversion." *American Economic Review*, 96(5): 1821-1834.

[9] **Chetty, Raj, Adam Looney, and Kory Kroft**. 2007. "Salience and Taxation: Theory and Evidence." National Bureau of Economic Research, Inc., NBER Working Papers: No. 13330.

[10] **Chetty, Raj**. 2009. "Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form Methods." National Bureau of Economic Research, Inc., NBER Working Papers: No. 14399.

[11] **Chetty, Raj and Emmanuel Saez.** 2009. "Teaching the Tax Code: Earnings Responses to an Experiment with EITC Recipients." National Bureau of Economic Research, Inc., NBER Working Papers: No. 14836.

[12] **de Bartolome, Charles**. 1995. "Which Tax Rate Do People Use: Average or Marginal?" *Journal of Public Economics*, 56: 79-96.

[13] **DellaVigna, Stefano**. 2007. "Psychology and Economics: Evidence from the Field." National Bureau of Economic Research, Inc., NBER Working Papers: No. 13420.

[14] **Feldman, Naomi E. and Peter Katuščák**. 2006. "Should the Average Tax Rate be Marginalized?" CERGE Working Paper No. 304.

[15] **Finkelstein, Amy N**. 2007. "E-Z Tax: Tax Salience and Tax Rates." National Bureau of Economic Research, Inc., NBER Working Papers: No. 12924.

[16] **Gabaix, Xavier and David Laibson**. 2006. "Shrouded Attributes, Consumer Myopia, and Information Suppression in Competitive Markets." *Quarterly Journal of Economics*, 121(2): 505-540.

[17] **Gallagher, Kelly and Erich Muehlegger**. 2008. "Giving Green to Get Green: Incentives and Consumer Adoption of Hybrid Vehicle Technology." Harvard KSG Working Paper.

[18] **Green, Jerry and Daniel Hojman**. 2009. "Choice, Rationality, and Welfare Measurement." Harvard University mimeo.

[19] **Harberger, Arnold C**. 1964. "The Measurement of Waste." *American Economic Review*, 54(3): 58-76.

[20] **Hausman, Jerry A. and Whitney K. Newey**. 1995. "Nonparametric Estimation of Exact Consumers Surplus and Deadweight Loss." *Econometrica*, 63(6): 1445-1476.

[21] **Kerschbamer, Rudolf and Georg Kirchsteige**r. 2000. "Theoretically robust but empirically invalid? An experimental investigation into tax equivalence." *Economic Theory*, 16: 719-734.

[22] **Kotlikoff, Laurence J. and Lawrence H. Summers**. 1987. "Tax Incidence," in *Handbook of Public Economics* Vol. 2, ed. Alan J. Auerbach and Martin Feldstein, 1043-1092. Amsterdam: Elsevier Science Publishers B. V.

[23] **Liebman, Jeffrey B. and Richard J. Zeckhauser**. 2004. "Schmeduling." Harvard KSG Working Paper.

[24] **Mazar, Nina, Botond Koszegi, and Dan Ariely**. 2008. "Price-Sensitive Preferences." UC-Berkeley Working Paper.

[25] **Mill, John S**. 1848. *Principles of Political Economy.* Oxford: Oxford University Press.

[26] **Mirrlees, James A**. 1971. "An Exploration in the Theory of Optimum Income Taxation." *The Review of Economic Studies*, 38(2): 175-208.

[27] **Mohring, Herbert**. 1971. "Alternative welfare gain and loss measures." *Western Economic Journal*, 9: 349-368.

[28] **Ramsey, Frank P**. 1927. "A Contribution to the Theory of Taxation." *Economic Journal*, 37(145): 47-61.

[29] **Slemrod, Joel B**. 2006. "The Role of Misconceptions in Support for Regressive Tax Reform." *National Tax Journal*, 59(1): 57-75.

Appendix A: Proofs

Proposition 4 nests the results in Propositions 2 and 3 as the case where $t_0^E = 0$ and $t_0^S = 0$. The proof of Proposition 4 below provides approximations for EB that are accurate for small initial tax rates and tax changes. To be precise, let $t_0^E = \lambda \bar{t}^E$, $t_0^S = \lambda \bar{t}^S$, and $\Delta t = \lambda \overline{\Delta t}$. I derive expressions for $EB(\Delta t | t_0^S, t_0^E)$ using Taylor expansions that ignore terms proportional to λ^n where $n \geq 3$. That is, the formulas ignore terms proportional to $(\Delta t)^3$, $(t_0^S)^3$, $(\Delta t)^2 t_0^S$, $(\Delta t)^2 t_0^E$, $(\Delta t)(t_0^E)^2$, etc.

Proof of Proposition 4i

Definitions: Let $x_0^E = x(p+t_0^E, 0)$, $x_0 = x(p+t_0^E, t_0^S)$, $x_1 = x(p+t_0^E, t_0^S+\Delta t^S)$, $x_0^* = x(p+t_0^E+t_0^S, 0)$ and $x_1^* = x(p+t_0^E+t_0^S+\Delta t^S, 0)$. Let $V^*(p+t_0^E, t_0^S, Z)$ denote the utility attained by a fully optimizing agent who consumes the optimal bundle $(x^*(p+t^E, t^S, Z), y^*(p+t^E, t^S, Z))$. Let $R^*(p+t^E, t^S, Z) = (t^E+t^S)x^*(p+t^E, t^S, Z)$ denote tax revenue obtained from a fully optimizing agent.

Let the agent's loss from failing to optimize relative to the tax be denoted by

$$G(t_0^E, t_0^S) = e(p, 0, V^*(p+t_0^E, t_0^S)) - e(p, 0, V(p+t_0^E, t_0^S))$$

The gain in revenue due to the agent's under-reaction to the tax is

$$\Delta R(t_0^E, t_0^S) = R(p+t_0^E, t_0^S, Z) - R^*(p+t_0^E, t_0^S, Z)$$

Recall that excess burden in the full optimization case is

$$EB^*(t_0^E, t_0^S) = Z - e(p, 0, V(p+t_0^E, t_0^S, Z)) - R^*(p+t_0^E, t_0^S, Z).$$

Combining these three equations, I rewrite the formula for excess burden in (3) as

$$EB(t_0^E, t_0^S) = EB^* - \Delta R + G.$$

Using this formulation for EB, the excess burden of a sales tax increase Δt is

$$EB(\Delta t | t_0^E, t_0^S) = \Delta EB^* - \Delta\Delta R + \Delta G \tag{9}$$

where the difference operator $\Delta X(t_0^E, t_0^S) = X(t_0^E, t_0^S+\Delta t) - X(t_0^E, t_0^S)$. I will use Taylor expansions to obtain simple expressions for each of these three terms below.

i) Auerbach (1985) shows that ignoring third-order terms, excess burden for an optimizing agent is approximately

$$\Delta EB^* = -\frac{\partial x^c}{\partial p}\Delta t[t_0^E + t_0^S + \frac{1}{2}\Delta t]$$

ii) The $\Delta\Delta R$ term can be written as:

$$\Delta\Delta R = -(t_0^E + t_0^S + \Delta t)(x_1^* - x_1) + (t_0^E + t_0^S)(x_0^* - x_0)$$

Ignoring the third- and higher-order terms (proportional to $\lambda^n, n \geq 3$) in EB, I write this equation as

$$\Delta\Delta R = (\Delta t)^2 \left(\frac{\partial x}{\partial t} - \frac{\partial x}{\partial p}\right) + \Delta t\left(\frac{\partial x}{\partial t} - \frac{\partial x}{\partial p}\right)(t_0^E + 2t_0^S)$$

iii) Simplifying the expression for G requires more work. First recall that the expenditure function is

$$e(p, t^S, V) = (p + t^S)x^c(p, t^S, V) + y^c(p, t^S, V)$$

and hence

$$\frac{\partial e}{\partial V} = (p + t^S)\frac{\partial x^c}{\partial V} + \frac{\partial y^c}{\partial V}.$$

The expenditure minimization problem is

$$\min(p + t^S)x^c + y^c \text{ s.t. } u(x) + v(y) = V$$

Differentiating the utility constraint for the expenditure minimization problem (EMP) yields

$$u'(x^c)\frac{dx^c}{dV} + v'(y^c)\frac{dy^c}{dV} = 1$$

The first-order condition for the EMP implies

$$u'(x^{*c}) = (p + t^S)v'(y^{*c})$$

and hence

$$(p + t_0^E + t_0^S)\frac{\partial x^{*c}}{\partial V} + \frac{\partial y^{*c}}{\partial V} = \frac{1}{v'(y^{*c})} = \frac{\partial e(p + t_0^E, t_0^S, V^*)}{\partial V}$$

where all the derivatives are evaluated at $(p + t_0^E, t_0^S, V^*)$. Using a Taylor expansion, it follows that

$$G = \frac{\partial e(p + t_0^E, t_0^S, V^*)}{\partial V}[V^*(p+t_0^E, t_0^S, Z) - V(p+t_0^E, t_0^S, Z)] - \frac{1}{2}\frac{\partial^2 e(p + t_0^E, t_0^S, V^*)}{\partial V^2}[V^* - V]^2 + ...$$

I show below that $V^* - V$ is proportional to λ^2; hence, the $[V^* - V]^2$ and higher-order terms in this expansion can be ignored under the second-order approximation. Hence, one can write

$$G = \frac{[V^*(p + t_0^E, t_0^S, Z) - V(p + t_0^E, t_0^S, Z)]}{v'(y^{*c}(p + t_0^E, t_0^S, V^*))}$$

Define the utility gain from choosing the optimal level x^* vs. another point x as

$$\begin{aligned}\widetilde{G}(x) &= u(x^*) - u(x) + v(y^*) - v(y) \\ &= u'(x^*)(x^* - x) - \frac{1}{2}u''(x^*)(x^* - x)^2 + O_u^3 + v'(y^*)(y^* - y) - \frac{1}{2}v''(y^*)(y^* - y)^2 + O_v^3\end{aligned}$$

where O_u^3 and $+O_v^3$ represent the third- and higher order terms of the Taylor expansions for u and v. All of the terms in O_u^3 and $+O_v^3$ are ultimately proportional to λ^n where $n \geq 3$, so

I ignore these terms from this point onward.

Using the first-order condition that characterizes the choice of the fully-optimizing agent,

$$u'(x^*) = (p + t_0^E + t_0^S)v'(y^*)$$

and the identity

$$(p + t_0^E + t_0^S)(x^* - x) = (y - y^*)$$

one obtains

$$\begin{aligned} \widetilde{G} &= -\frac{1}{2}u''(x^*)(x^* - x)^2 - \frac{1}{2}v''(y^*)(y^* - y)^2 \\ &= -\frac{1}{2}(x^* - x)^2[u''(x^*) + v''(y^*)(p + t_0^E + t_0^S)^2] \end{aligned} \tag{10}$$

Totally differentiating the fully-optimizing agent's first-order condition with respect to p yields

$$\begin{aligned} u''(x^*)\frac{\partial x^*}{\partial p} &= v'(y^*) + (p + t_0^E + t_0^S)v''(y^*)\frac{\partial y^*}{\partial p} \\ &= v'(y^*) + (p + t_0^E + t_0^S)[-(p + t_0^E + t_0^S)\frac{\partial x^*}{\partial p} - x^*]v''(y^*). \end{aligned}$$

It follows that

$$[u''(x^*) + (p + t_0^E + t_0^S)^2 v''(y^*)]\frac{\partial x^*}{\partial p} = v'(y^*) - (p + t_0^E + t_0^S)x^* v''(y^*)$$

and hence

$$\widetilde{G} = -\frac{1}{2}(x^* - x)^2 \frac{[v'(y^*) - (p + t_0^E + t_0^S)x^* v''(y^*)]}{\partial x^*/\partial p}. \tag{11}$$

Defining $\gamma_y = -y^* v''(y^*)/v'(y^*)$ it follows that

$$G \simeq \frac{\widetilde{G}}{v'(y^*)} = -\frac{1}{2}(x^* - x)^2 \frac{1}{\partial x^*/\partial p}[1 + (p + t_0^E + t_0^S)\frac{x^*}{y^*}\gamma_y]. \tag{12}$$

Finally, I use a result from Chetty (2006) which relates the coefficient of relative risk aversion to the ratio of the income effect to the substitution effect:

$$\gamma_y = \frac{-y^*}{p + t_0^E + t_0^S}\frac{\frac{\partial x^*}{\partial z}}{\frac{\partial x^{*c}}{\partial p}}. \tag{13}$$

Inserting this expression into (12) yields

$$G \simeq -\frac{1}{2}(x^* - x)^2 \frac{1}{\partial x^*/\partial p}[1 - x^* \frac{\frac{\partial x^*}{\partial z}}{\frac{\partial x^{*c}}{\partial p}}] = -\frac{1}{2}(x^* - x)^2 \frac{1}{\partial x^{*c}/\partial p}$$

Note that $\partial x^c/\partial p = \partial x^{*c}/\partial p$ and $\partial x/\partial p = \partial x^*/\partial p$ at the no-sales-tax point $p + t_0^E$ under A2. Thus, for small Δt, t_0^S, and t_0^E,

$$\begin{aligned}\Delta G &\simeq -\frac{1}{2}\frac{1}{\partial x^c/\partial p}\{(x_1^* - x_1)^2 - (x_0^* - x_0)^2\} \\ &\simeq -\frac{1}{2}\frac{1}{\partial x^c/\partial p}(\Delta t)^2\{(\frac{\partial x}{\partial p})^2 + (\frac{\partial x}{\partial t})^2 - 2\frac{\partial x}{\partial t}\frac{\partial x}{\partial p}\} - \Delta t\frac{1}{\partial x^c/\partial p}(\frac{\partial x}{\partial p} - \frac{\partial x}{\partial t})^2 t_0^S\end{aligned}$$

where the second approximation ignores third- and higher-order terms and all derivatives are evaluated at $(p + t_0^E, 0)$.

Combining the expressions for ΔG, $\Delta\Delta R$, and ΔEB^* above using (9) and collecting terms yields the formula in Proposition 4ii.

Proof of Proposition 4ii

Definitions: Let $p_0^E = p(t^E, 0)$, $p_0 = p(t^E, t^S)$, and $p_1 = p(t^E, t^S + \Delta t)$. Let π_0 and π_1 denote profits before and after implementation of the tax. To reduce notation, I suppress wealth in the demand function and write $x(p,t)$ since Z does not affect x when utility is quasilinear. Let $x_0^E = x(p_0^E + t_0^E, 0)$, $x_0 = x(p_0 + t_0^E, t_0^S)$, $x_1 = x(p_1 + t_0^E, t_0^S + \Delta t^S)$. Let $R_0 = (t_0^E + t_0^S)x_0$ and $R_1 = (t_0^E + t_0^S + \Delta t^S)x_1$.

Excess burden with pre-existing taxes and quasi-linear utility is (Auerbach 1985):

$$EB(\Delta t|t_0^S, t_0^E) = Z - e(p_0 + t_0^E, t_0^S, V(p_1 + t_0^E, t_0^S + \Delta t, Z + \pi_1)) + (\pi_0 - \pi_1) - (R_1 - R_0) \quad (14)$$

Using the definition of the expenditure function for the quasilinear case and the definition of the profit function, I write (14) as

$$EB = u(x_0) - u(x_1) + c(x_1) - c(x_0) \quad (15)$$

This expression measures the area of the trapezoid that lies between the price-demand and supply curves between x_0 and x_1, shown in Appendix Figure 1. The derivation below is essentially an algebraic calculation of the area of that trapezoid using a series of Taylor expansions. To begin, I write (15) as

$$EB = u'(x_0)(x_0 - x_1) - \frac{1}{2}u''(x_0)(x_0 - x_1)^2 - O_u^3 + c'(x_0)(x_1 - x_0) + \frac{1}{2}c''(x_0)(x_1 - x_0)^2 + O_c^3$$

where $O_f^3 = \sum_3^\infty \frac{f^n(x_0)}{n!}(x_1 - x_0)^n$, $f = u, c$ consists of the third- and higher-order elements of the Taylor series. Observe that the first-order condition for the optimal choice of x for the consumer is

$$u'(x^*(p)) = p$$

Total differentiation of this condition yields

$$u''(x^*(p)) = \frac{1}{\partial x^*(p)/\partial p}$$

Recognizing that $x^*(p_0^E + t_0^E, 0) = x(p_0^E + t_0^E, 0) = x_0^E$, I take a Taylor approximation around x_0^E to write

$$\begin{aligned} u'(x_0) &= p_0^E + t_0^E + u''(x_0^E)(x_0 - x_0^E) + ... \\ u''(x_0) &= \frac{1}{\partial x/\partial p} + u'''(x_0^E)(x_0 - x_0^E) + ... \\ x_0 - x_0^E &= \frac{dx}{dt^S} t_0^S + \frac{1}{2} \frac{dx}{d(t^S)^2} (t_0^S)^2 + ... \end{aligned}$$

Note that the derivatives in these equations are evaluated at the no-sales-tax equilibrium $(p_0^E + t_0^E, 0)$ because this is the only point at which the first-order conditions hold.

Similarly, the first-order conditions from firm optimization and a Taylor approximation around x_0^E can be used to write

$$\begin{aligned} c'(x_0) &= p_0^E + c''(x_0^E)(x_0 - x_0^E) \\ c''(x_0) &= \frac{1}{\partial S/\partial p} + c'''(x_0^E)(x_0 - x_0^E) + ... \end{aligned}$$

Finally, a Taylor expansion around x_0 yields:

$$(x_0 - x_1) = \frac{dx}{dt^S} \Delta t - \frac{1}{2} \frac{d^2 x}{d(t^S)^2} (\Delta t)^2 + ...$$

Ignoring the third- and higher-order terms (proportional to $\lambda^n, n \geq 3$) in EB, I can combine the Taylor expansions above to write

$$\begin{aligned} EB &= -\frac{1}{2}(\frac{1}{\partial x/\partial p} - \frac{1}{\partial S/\partial p})(\frac{dx}{dt^S}\Delta t)^2 \\ &\quad -(\frac{dx}{dt^S}\Delta t - \frac{1}{2}\frac{d^2x}{d(t^S)^2}(\Delta t)^2)(t_0^E + \frac{dx}{dt^S}(\frac{1}{\partial x/\partial p} - \frac{1}{\partial S/\partial p})t_0^S) \end{aligned} \tag{16}$$

To simplify this expression, I use the expression for $\frac{\partial p}{\partial t^S}$ in Proposition 1 to obtain the following result:

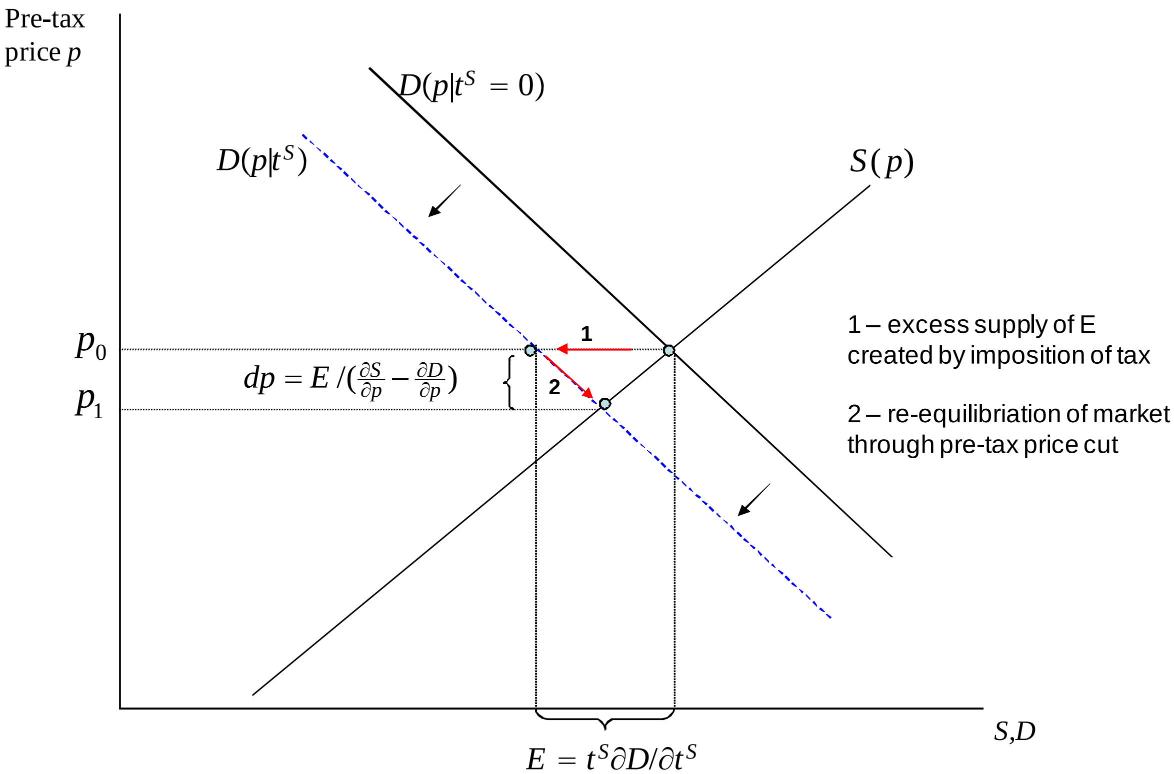
$$\frac{dx}{dt^S}(\frac{1}{\partial x/\partial p} - \frac{1}{\partial S/\partial p}) = (\frac{\partial x}{\partial p}\frac{\partial p}{\partial t^S} + \frac{\partial x}{\partial t^S})(\frac{1}{\partial x/\partial p} - \frac{1}{\partial S/\partial p}) = \theta \tag{17}$$

Combining (17) with (16) gives

$$EB = -\frac{1}{2}(\Delta t)^2 \theta \frac{dx}{dt^S} - \frac{dx}{dt^S}\Delta t(t_0^E + \theta t_0^S).$$

Figure 1

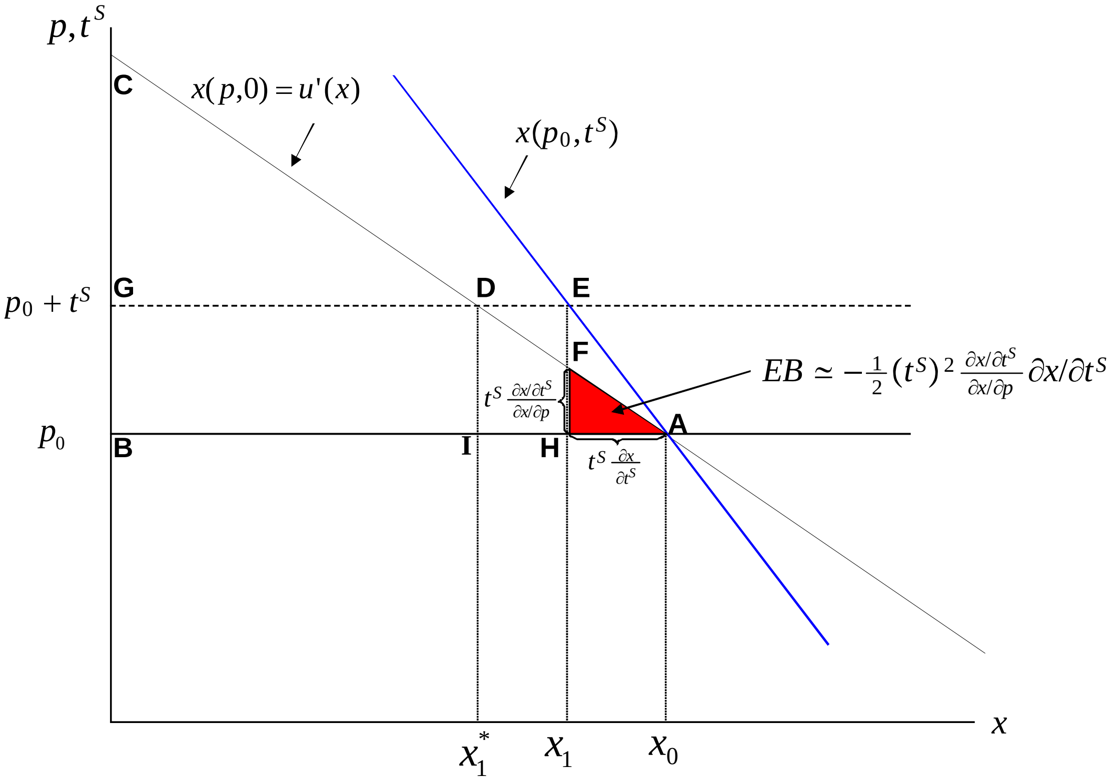
Incidence of Taxation



NOTE–This figure illustrates the incidence of introducing a tax t^s levied on consumers in a market that is initially untaxed. The figure plots supply and demand as a function of the pre-tax price p. The initial price-demand curve is $D(p|t^S = 0)$; the price-demand curve after the tax is introduced is $D(p|t^S)$. When the tax is levied, the demand curve shifts inward by $t^S \times \partial D/\partial t^S$ units, creating an excess supply of $E = t^S \times \partial D/\partial t^S$. To re-equilibriate the market, producers cut the pre-tax price by $E/(\partial S/\partial p - \partial D/\partial p)$ units, implying $\frac{dp}{dt^S} = \frac{\partial D/\partial t^S}{\partial S/\partial p - \partial D/\partial p}$.

Figure 2

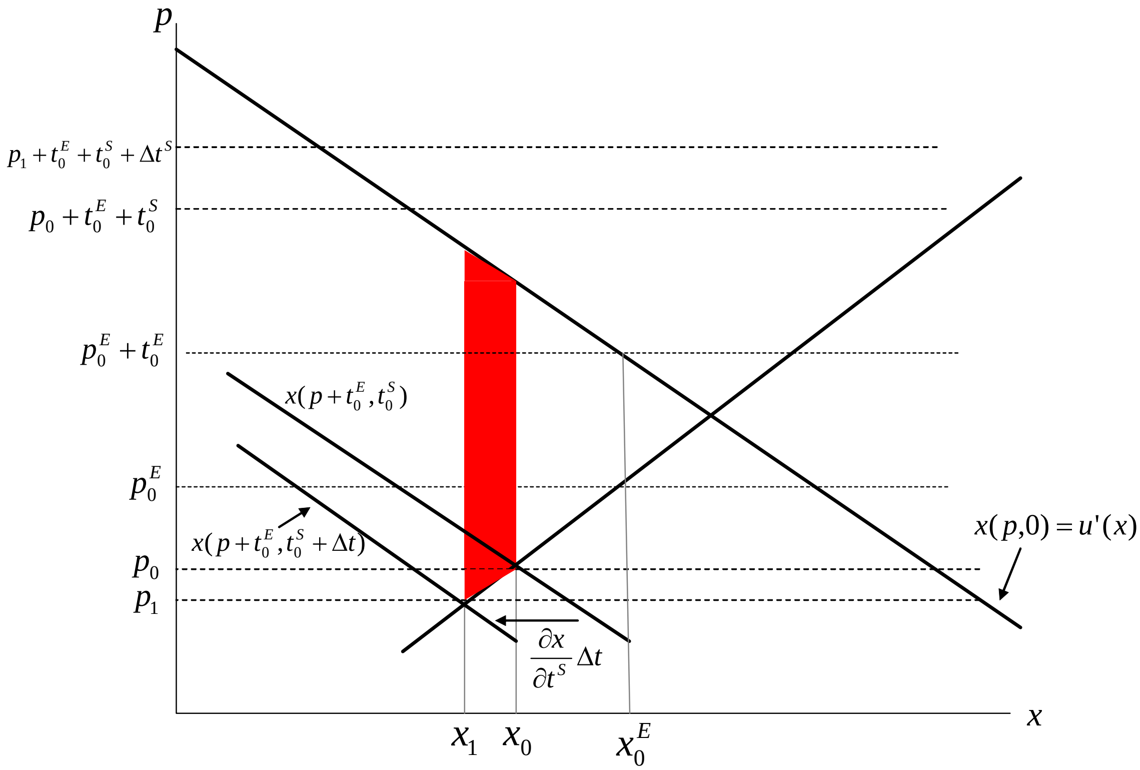
Excess Burden with No Income Effect for Good x ($\frac{\partial x}{\partial Z} = 0$)



NOTE–This figure illustrates the deadweight cost of introducing a tax t^s levied on consumers when $\frac{\partial x}{\partial Z} = 0$ and producer prices are fixed. The figure plots two demand curves: (1) the price-demand curve $x(p, 0)$, which shows how demand varies with the pre-tax price of the good and (2) the tax-demand curve $x(p_0, t^S)$, which shows how demand varies with the tax. The figure is drawn assuming $|\partial x/\partial t^S| \leq |\partial x/\partial p|$, consistent with existing empirical evidence. The tax reduces demand from x_0 to x_1. The consumer's surplus after the implementation of the tax is given by triangle *DGC* minus triangle *DEF*. The revenue raised from the tax corresponds to the rectangle *GBEH*. The change in total surplus – government revenue plus consumer surplus – equals the shaded triangle *AFH*.

Appendix Figure 1

Excess Burden of Taxation with Pre Existing Taxes



NOTE–This figure depicts the excess burden of increasing the sales tax by Δt starting from initial tax rates of (t_0^E, t_0^S) when $\frac{\partial x}{\partial Z} = 0$ and prices are endogenous (Proposition 4ii). The figure plots three Marshallian demand curves as a function of the pre-tax price: (1) $x(p, 0)$ – the price-demand curve absent taxes, which allows us to recover true preferences; (2) $x(p + t_0^E, t_0^S)$ – the initial demand curve prior to the tax increase; and (3) $x(p + t_0^E + \Delta t, t_0^S)$ – the demand curve after the tax increase. The figure also depicts demand with only a pre-existing excise tax $x_0^E = x(p_0^E + t_0^E, 0)$, which is the point from which the second-order approximations are made to calculate the area of the trapezoid.